Firms' Choices of Wage-Setting Protocols[*]

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Abstract

We study a labor market characterized by search frictions in which firms choose between posting non-negotiable wage offers and bargaining wages with individual workers. We use the model to study the positive and normative implications of heterogeneous wage-setting strategies in labor markets, as well as the potential effect of policies that seek to regulate wage-setting. We analytically derive - and empirically validate - a testable prediction from the model regarding the cross-sectional prevalence of bargaining and renegotiation of wages among workers. We then estimate the model and use it to evaluate counterfactuals in which either wage-setting procedure is mandated. We find that eliminating bargaining reduces the overall gender gap in wages by 6%, the education gap by 3%, and residual wage dispersion by 12%, while leading to welfare losses for workers. Similar numbers are observed when bargaining is mandated, with ensuing welfare gains for workers. Either policy raises output by 1-3% by eliminating inefficient job mobility, but accounting for firm responses in vacancy creation can overturn these effects.

1 Introduction

The manner in which wages are set is not the same for all workers. We investigate the impact of heterogeneity in wage-setting procedures on labor market efficiency and wage distributions, as well as the determination of firms' choices of these procedures in equilibrium. In order to do so, we formulate and estimate a model in which firms choose to either bargain with workers or to post non-negotiable wage contracts at the time that they create a job vacancy. We derive clear cross-sectional predictions for rates of bargaining and wage renegotiation and validate the model's mechanisms by verifying these predictions in data. We then extend the model to allow for heterogeneity in productivity across workers and jobs and examine the empirical content of equilibrium wage-setting in this case. We estimate the extended model and use it to evaluate the contribution of differences in wage-setting strategies across and within markets to wage inequality, gaps in wages by gender and education, and the misallocation of workers across jobs.

Our research is motivated by the observation that bargaining and wage-posting are both common methods of wage-setting in the labor market (Hall and Krueger, 2012) with potentially quite different implications for wage inequality. While posted wages should typically reflect only the characteristics of a particular job, bargaining introduces the worker's

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outside option as an influence on wages, thereby contributing to inequality among equally productive employees. In the presence of search frictions, identical workers receive different wages because of differences in the quality of the jobs they are able to find. This phenomenon, often referred to as frictional or residual wage dispersion, introduces luck as a relevant factor in the wage distribution. In the particular case of bargaining, workers carry their good or bad fortunes with them to their next job[1] because it represents their outside option in wage negotiations. In this way, luck tends to be more persistent in labor markets with bargaining and renegotiation, and exacerbates frictional wage dispersion. For the purposes of policy and prediction, it is important to understand how differences in the prevalence of bargaining contribute to wage inequality in equilibrium. Furthermore, since Hall and Krueger (2012) document heterogeneity in wage-setting among observationally equivalent workers, this question is relevant for the determination of both between and within-group wage inequality.

Since bargaining creates a link between workers' outside options and current wages (that may themselves be used as the outside option for setting future wages) one particular concern is that this creates a cascading cycle of disadvantage for workers who have traditionally faced discrimination in the labor market. In response to such concerns, salary history bans (which make it illegal for firms to inquire about pay at previous jobs and for previous employers to supply such information) have emerged as a popular policy initiative. As of August 2020, 19 states[2] have introduced salary history bans, some of which cover all employers, while others are limited to public employers only. The rapid emergence of policies such as these that seek to regulate the wage-setting process highlights the need for a labor market model that allows for counterfactual policy evaluation when wage-setting strategies are endogenously determined in equilibrium. This applies broadly to all labor market interventions in which we do not expect wage-setting to be policy invariant, following traditional critiques (Marschak, 1974; Lucas, 1976).

Adding to the importance of modeling firms' choices of wage-setting protocols is the fact that the propensity to bargain exhibits strong patterns by gender and education. It is well documented that women are substantially less likely to bargain than men (Babcock and Laschever, 2009; Hall and Krueger, 2012), and recent empirical work has sought to estimate how differences in bargaining contribute to the gender gap in wages. Card, Cardoso, and Kline (2015) analyze a large Portuguese administrative data set and conclude that bargaining ability differences account for 5 to 15 percent of wage differences in that labor market. Dittrich, Knabe, and Leipold (2014) carry out a face-to-face bargaining experiment in which males and females interact with one another while alternatively playing the roles of firm owner and worker. They find no differences in outcomes by gender when playing the role of the owner, but that women do significantly worse than men when bargaining as a worker. Biasi and Sarsons (2021) find that switching to flexible pay led to widening gender gaps among Wisconsin public school teachers. Flinn, Todd, and Zhang (2020) estimate a partial equilibrium search model and find that bargaining power differences are the largest driver of the gender gap in wages. Due to this body of evidence, regulating wage-setting is seen as a potential avenue to reduce gender gaps in wages. Our model provides an estimate of the contribution of bargaining and renegotiation to gender gaps as well as a tool for counterfactual evaluation of such policies in equilibrium. We emphasize that while the model can replicate these empirical findings, it also permits additional insights on the welfare and efficiency properties of different wage-setting regulations.

Hall and Krueger (2012) find that 35 percent of workers in their survey report having engaged in bargaining over their

[1] By next job, we are referring to the immediately following job in an uninterrupted employment spell. As is usually the case, we assume that once the individual enters unemployment, their outside option is simply the value of remaining in the unemployment state.

[2] The website HRDive.com keeps a running list of states with salary history bans: https://www.hrdive.com/news/salary-history-ban-states-list/516662/

wage, while 32 percent report that they knew exactly their potential wage prior to interview. These statistics bring to mind the two workhorse models of wage-setting in quantitative studies of frictional labor markets: bilateral bargaining with renegotiation, as studied by Cahuc, Postel-Vinay, and Robin (2006) and Dey and Flinn (2005), and the wage-posting model of Burdett and Mortensen (1998) and Albrecht and Axell (1984). Most strikingly, Hall and Krueger (2012) show that workers persistently exhibit heterogeneity in wage-setting procedures even when conditioning across a rich set of covariates. This fact suggests the potential existence of mixed wage-setting approaches *within* labor markets as well as across them. In our theoretical framework we allow firms to choose which of these canonical wage-setting procedures to associate with each vacancy. With an estimated version of the model we are able to match observed patterns of bargaining across and within groups as well as answer the important quantitative questions outlined above about the contribution of heterogeneous wage-setting to inequality.

The model also raises important and somewhat novel questions regarding labor market efficiency. Typical models of labor markets with search frictions overwhelmingly exhibit efficient job switches: workers will always move to jobs with higher productivities. With the exception of two related papers by Postel-Vinay and Robin (2004) and Doniger (2015) (discussed below), this is due to the assumption of a single, fixed, wage-setting protocol that generates a monotonic relationship between a job's flow output and private values. We will show in our setting that efficient mobility decisions are no longer guaranteed, due to the potential for competition between wage-posting and wage-bargaining vacancies. In particular, because posted wages are fixed and non-negotiable, a bargaining firm can outbid a more productive posting firm as long as they can produce at a higher rate of output than the wage offer of the posting firm. This phenomenon, which we refer to as inefficient mobility, leads to misallocation of workers across jobs within a labor market. This constitutes a final motivation for our study, which is to use our estimated model to quantitatively assess the loss in output that is created by heterogenous wage-setting through inefficient mobility.

Despite the clear importance of studying wage-setting choices for understanding labor market mobility, efficiency, and wage inequality, the overwhelming majority of studies assume environments in which the wage-setting protocol is fixed and homogenous. Before outlining our general contributions, it is worth discussing two important contributions that also have introduce endogenous wage-setting protocols. To the best of our knowledge, the first attempt to do so was Postel-Vinay and Robin (2004). Building on the framework of their earlier paper (Postel-Vinay and Robin, 2002), the authors introduce endogenous search intensity as the mechanism through which firms may wish to commit ex-ante to not match outside offers from other firms. They seek conditions on the underlying distribution of firm productivities that guarantee a separating equilibrium in which high productivity firms match counteroffers while low productivity firms do not.[3] Doniger (2015) uses a similar framework but relies on a cost differential rather than endogenous search effort to generate a separating equilibrium. The model, when estimated on German administrative data, can rationalize observed patterns in residual wage dispersion and labor share.

In our model we require firms to commit to a wage-setting protocol prior to the realization of the productivity associated with a particular job, so that by definition equilibria in our model are pooled. In doing so, we focus on differences across markets in the rates at which firms opt for bargaining and renegotiation over wage-posting, rather than on differences across firms within a market. Furthermore, we are able to show that mixed wage-setting equilibria exist even in markets

[3]It is important to note that their model does not produce inefficient mobility since the productivity of a match is determined by the product of the worker's type and the firm's type. Wage-posting firms make non-renegotiable offers that are increasing in their type, and the better firms engage in Bertrand-competition which ensures that the better firm wins all bargaining competitions. Since there is no idiosyncratic match heterogeneity in their model, there is no inefficient mobility in equilibrium.

where firms are identical to each other. The crucial mechanism in our model is somewhat different from the two papers that we have just discussed. The relative profitability of each protocol is determined by two main factors. The first is the share of the job surplus obtained by the worker when they bargain with a firm, which in the model is given by the parameter α. The second is the average number of job offers an employed worker receives before the job is destroyed for exogenous reasons, which is a market-level primitive we denote by κ.[4] In models of wage-posting, κ effectively determines the share of the job surplus captured by a posting firm. As κ increases, this reduces the profitability of posting versus bargaining, and so the equilibrium share of bargaining firms increases. We are able to verify this testable prediction by examining cross-sectional patterns across demographic groups.

Dynamics are crucial in our exercise, both for introducing the mechanism for endogenous bargaining choices and also for properly understanding wage data. In static settings, other work has shown that bargaining is preferable for firms in scenarios where the unobserved heterogeneity is sufficiently variable in the population. Michelacci and Suarez (2006) show that firms will prefer to bargain in markets where the variance in worker ability is sufficiently high due to the risk of posting a wage that cannot condition on this unverifiable characteristic. Since we allow for piece-rate wages in our model (that effectively condition wages on worker productivity), our model does not feature this mechanism. In recent work, Cheremukhin and Restrepo-Echavarria (2021) show that coexistence of wage-setting strategies can arise when search effort is endogenously directed, in the sense that workers can sample from any distribution of jobs according to some entropy cost relative to the population distribution. In this setting, sufficient heterogeneity in either search costs or match quality can result in mixed equilibria. While heterogeneity in match quality does influence the mixture of bargaining firms in our model, it is not the main mechanism of interest.

Building on some of the insights of prior work in this area, this paper seeks to make contributions in three broad areas. The first of these is to develop an analytically tractable model of heterogenous wage-setting, with simple rules that specify when a worker switches jobs and how wages are determined when bargained or renegotiated. These rules can be readily brought to a variety of empirical settings, including to matched employer-employee data. For example, Caldwell and Harmon (2019) have used a version of our model to estimate the effect of coworker networks on wages in Danish administrative data. Typical models of bargaining and renegotiation imply that these networks can influence wages by improving workers' outside options. However our hybrid approach is required to fit the data, since they find substantial differences in the influence of outside options across job types. In a similar spirit, Di Addario, Kline, Saggio, and Sølvsten (2020) use matched employer-employee data to estimate the effect among job switchers of prior ("origin") firms at their new ("destination") firms. They find substantial differences across sectors in the influence of origin firms, a fact that can be rationalized using the model developed in this paper. These studies highlight the value of a tractable quantitative framework such as ours that allows for mixed wage-setting methods in order to understand wage dynamics.

The paper's second contribution is to show how this combination of wage-setting strategies can be determined in equilibrium, when firms choose protocols, vacancies, and post wages to maximize profits. In the simplest case, when there is no difference in a worker's potential output across firms within the same market, we are able to derive a closed-form expression that determines the equilibrium fraction of each wage-setting protocol in that market. This expression specifically identifies one statistic as being informative about the rate of bargaining and renegotiation in a given market, κ. We combine data from Hall and Krueger (2012), the *Survey of Consumer Expectations* (SCE), and the *Current Population Survey* (CPS), to show that this statistic, as suggested by the model, does predict rates of both bargaining

[4] We refer to κ as a primitive for expositional purposes, but in the full model the rate of job offers obtained by employed and unemployed searchers is determined endogenously through firms' vacancy posting decisions.

and renegotiation across demographic groups. We then establish a set of results that show how equilibrium is determined when assumptions on match productivities are relaxed relative to this simple benchmark case, in particular allowing for heterogeneity in worker ability and job productivity. This version of the model exhibits inefficient mobility, which occurs when wage-posting firms are outbid by less productive bargaining firms. The importance of this phenomenon can be determined using the equilibrium wage offer function utilized by posting firms in equilibrium.

The paper's final contribution is to estimate a quantitative version of the model in order to explore the positive and normative implications of heterogeneous wage-setting in labor markets. We estimate the model with worker and job heterogeneity across segregated "markets," each of which consists of a group of workers belonging to the same birth cohort, gender, and schooling level. We find that wage dispersion due to search frictions explains 10 percent of the overall within-group variance in log wages, and 6 percent of overall between group variance. Model estimates also imply that 15 percent of all job moves are inefficient. We use the estimated model to evaluate two counterfactuals in which we alternatively mandate that each wage-setting protocol be used exclusively by firms. We find that either mandate reduces within group wage inequality by about 12 percent relative to the model baseline. We also find that eliminating bargaining reduces the gender gap in wages by 10 to 15 percent of the total gender gap across markets. In high education groups, the baseline model is able to attribute between 20 and 30 percent of the gender gap to search frictions, and so in the model the gap is halved for these groups. These counterfactuals reveal that a significant portion of the gender gap is attributable to differences in wage-setting, and that there exists some scope for policy intervention.

We also consider the effect of regulating wage-setting practices on welfare and output. We find that workers are made better off when bargaining and renegotiation is mandated by a regulator, and slightly worse off when posting is mandated. These results are robust to allowing for contact rates to be determined in equilibrium. With respect to output, both counterfactuals are able to eliminate inefficient mobility by forcing all firms to use the same wage-setting protocol. In doing so, output increases by between 1 and 3 percent overall. This particular finding however is not robust to allowing for endogenous contact rates. In particular, in our model with endogenous contact rates, bargaining mandates lead to a reduction in vacancy posting and an overall reduction in output. As a result, we find that forcing firms to post wages is most efficient when contact rates are determined endogenously.

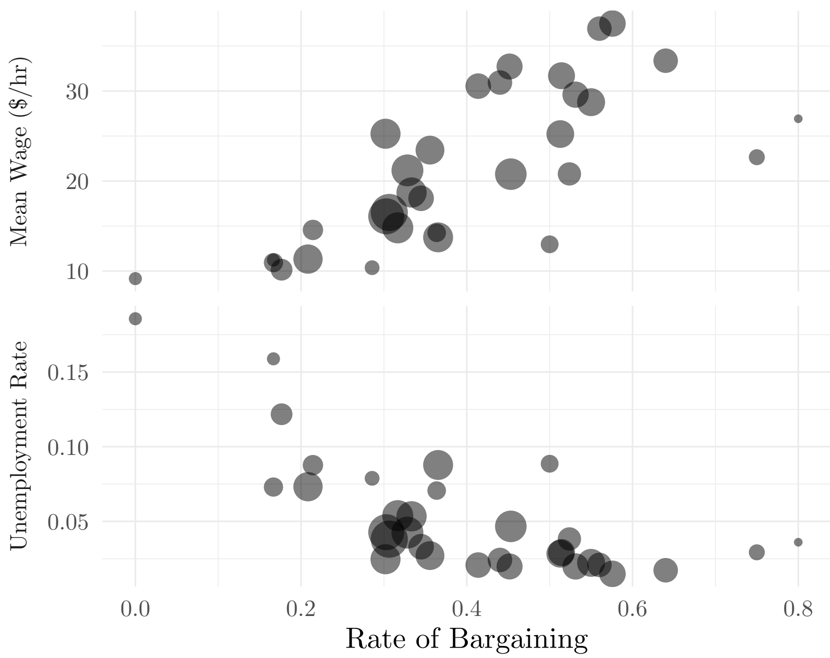
In order to provide some empirical context for the model developed below, we first use the survey data collected by Hall and Krueger (2012) to fix ideas about the prevalence of bargaining across markets. One question in particular from that survey indicates whether workers bargain over their wage.[5] Figure 1 calculates rates of reported bargaining by groups defined in terms of birth cohort, gender, and education, and matches these to corresponding rates of unemployment and mean wages in the CPS (Appendix A provides explicit details). Interpreting wages and unemployment as proxies of labor demand, Figure 1 demonstrates that bargaining positively correlates with labor demand in the cross-section.

Figure 1 suggests that matching this general pattern is a natural criterion for success for any model seeking to understand differences in wage-setting practices across labor markets. Our model provides an equilibrium relationship between wage-setting protocols and an alternative proxy for labor demand, κ. It is through this mechanism that our model will match these important cross-sectional facts. When firms post vacancies to maximize profits, more productive markets will feature higher contact rates, which leads to higher rates of bargaining and renegotiation. These markets will also feature higher wages and lower rates of unemployment.

In Appendix A we plot the same figure using a measure of posting taken from the same data, finding an inverse

[5] Appendix A.1 provides explicit details on the phrasing of questions regarding bargaining and wage-posting and a discussion of the issues surrounding their interpretation.

Figure 1: The Relationship Between Bargaining and Wages/Unemployment



This figure shows the relationship between wages and unemployment and rates of bargaining found in the survey of Hall and Krueger (2012). Each point represents a combination of age category, sex, and education category. Age categories are 20-29, 30-39, 40-49, and 50-59. Education categories are high school dropouts, high school graduates or equivalent, less than four years of college, and four or more years of college. Rates by demographic cell are matched to labor market statistics calculated using the *Current Population Survey* (CPS). Appendix A provides additional details. Points are proportional to sample size.

relationship. This suggests that bargaining and wage posting are directly competing alternatives. We do not emphasize patterns on posting due to the interpretation of this question being slightly more ambiguous (see Appendix A for a discussion), but the model we study does build in this pattern by assuming that when a vacancy is created, wages can be set either by bargaining (and later renegotiating) or by posting a non-negotiable wage, both of which are conditional on the worker's potential productivity at the firm. The model ties the ability to renegotiate with the ability to bargain. This choice can be justified *ex-ante* by the fact that it allows the theory to combine two canonical methods of wage-setting in the literature: wage posting (Burdett and Mortensen, 1998; Albrecht and Axell, 1984) and bargaining with renegotiation (Cahuc et al., 2006; Dey and Flinn, 2005). We will see that *ex-post* the choice is justified by the model's ability to match patterns in both bargaining and renegotiation across demographic groups, a feature enabled by their forced coupling in our framework. For the sake of robustness we also consider equilibria in which additional wage-setting protocols can be chosen. In doing so we find that allowing firms to either (1) renegotiate initially posted wages; or (2) post efficient contracts as in Stevens (2004) crowds out all other wage-setting strategies and results in counterfactual equilibria in which no workers report bargaining over their initial wage. As a result, we restrict attention to fixed wage contracts that are potentially renegotiated given a change in a worker's outside option.[6]

[6]That is, if the job is one in which wages are posted, the firm will never renegotiate the wage. However, if the job is one in which wages are

In the next section (Section 2) we illustrate the workings of the model by considering a simplified version in which all worker-firm pairs within the same (segmented) labor market produce the same output. In this case, we are able to derive an analytic expression for the equilibrium fraction of bargaining firms in the market. The expression shows that rates of bargaining and renegotiation across markets are driven by two key primitives: κ and α.[7] While the latter is not directly measurable, the former can be proxied by observed rates in the data. We combine data from the CPS, the SCE, and Hall and Krueger (2012) to show that this empirical proxy [8] does robustly predict rates of bargaining and renegotiation across demographic groups. We take this as validation of the model's key mechanism for explaining rates of bargaining and renegotiation across groups.

Next, in Section 3 we enrich the model to allow for heterogeneity in production across workers and firms. We derive simple rules for worker mobility and bargained wages, taking as given both the fraction of firms that choose to bargain and the distribution of wage offers from posting firms. This analysis produces a simple statistic that is sufficient for determining worker mobility outcomes, the maximum attainable wage at a firm. Using the distribution of this statistic in the labor market, we are able to characterize wage offers from both types of firms and the rate at which firms choose to bargain is determined in equilibrium. In Section 4 we discuss identification of the model and outline our approach to estimation. We provide an analysis of the general properties of our estimated model in Section 5. In Section 6 we examine the contribution of firms' wage-setting choices to several measures of wage inequality, as well as the potential scope for policies that regulate wage-setting by forcing firms to uniformly adopt only one wage-setting protocol. Both counterfactuals eliminate inefficient job-to-job mobility decisions, allowing us to quantify the output cost of inefficient mobility in the baseline economy. In Section 7 we conclude by discussing limitations of the model and potential avenues for future research.

2 A Simple Model of Equilibrium Wage-Setting Protocols

In their seminal paper, Burdett and Mortensen (1998) show that even when output is homogenous within a labor market, search frictions can generate equilibrium wage dispersion. This section provides an extension of that framework in which firms additionally choose whether to post wages in this fashion, or engage in bargaining with renegotiation as in Dey and Flinn (2005) and Cahuc et al. (2006). We will adopt throughout the notational convention that for any distribution function F, $\tilde{F}(x) = 1 - F(x)$ denotes the survivor function. Appendix B provides additional details on all model derivations.

2.1 Environment

Market Primitives Time is continuous, and the economy is populated by a unit mass of workers and vacancy-owning firms, both of whom are risk neutral and discount the future at a rate ρ. Production can only occur when workers match with vacant jobs through undirected search. A matched worker-firm pair produces a flow output z, which is the same for all matches in a market. All jobs are destroyed at an exogenous rate δ. When jobs are destroyed, workers return to unemployment and firms receive the value of their vacancy, which is set to zero due to a free entry condition.

Jobs are distinguished by one key characteristic, which is the type of wage-setting protocol tied to the vacancy, which

bargained, an increase in the worker's outside option will result in a wage increase.

[7] In the general equilibrium version of the model with endogenous contact rates κ is not a primitive since λ_E is not exogenous. In the partial equilibrium version of the model it is.

[8] The observed rate of employer-employer (EE) transitions divided by the rate of employment to unemployment (EU) transitions.

we denote by $B \in \{R, N\}$, where $B = N$ indicates that the wage is determined through posting and $B = R$ indicates that it is determined through negotiation. Thus, workers can be in one of three employment states: unemployed, employed at an R-firm, or employed at an N-firm. Unemployed workers have a flow utility equal to b, and match with vacant jobs at the Poisson rate, λ_U. Employed workers have a flow utility equal to their wage, w, and match with other vacancies at a rate $\lambda_E < \lambda_U$. Given that search is undirected, the fraction of vacancies posted by R-firms, p_R, is also the fraction of R-firms that are met by workers. Firms value only the profit from each match, defined as output net of wages, $z - w$. Finally, we define κ, an important market primitive as

$$\kappa = \frac{\lambda_E}{\delta}.$$

As in many models with on-the-job search, we will find that κ is crucial for determining the distribution of workers over employment states and equilibrium behavior.

Wage-Setting Protocols When $B = N$, the wage is posted and is therefore *non-negotiable* and fixed throughout the duration of the match. When $B = R$, this indicates that the wage will be *bargained*, using the worker's current outside option, and is subject to *renegotiation* if the worker's outside option improves. Importantly, as in Cahuc et al. (2006) and Dey and Flinn (2005), if a worker at an R-firm meets another R-firm, this induces Bertrand competition between the firms resulting in the worker claiming the full value of the match with wage z in this simple environment with homogeneous productivity.

2.2 Model Solution

Worker Values To clarify the dynamics of wages and mobility in the model, it will be useful to write the recursive formulation of the worker's value function, beginning with unemployment. At rate λ_U they meet a firm, and with probability p_R at that firm the wage will be determined by bargaining. In this instance, the worker will receive the value of unemployment, V_U, plus a fraction α of the surplus. This surplus is defined as the total value of the match, T, minus the value of unemployment. With probability $1 - p_R$, the worker meets a firm in which wages are posted, with the wage offer drawn from the equilibrium distribution of posted wages Φ. In this case they will receive the value $V_N(w)$. We then can write the flow value of unemployed search as:

$$\rho V_U = b + \lambda_U(1 - p_R) \int_{w^*} (V_N(w) - V_U) d\Phi(w) + \lambda_U p_R \alpha(T - V_U)$$

where the total value of the match can be written as:

$$(\rho + \delta)T = z + \delta V_U$$

and w^* is the reservation wage for the worker, characterized by

$$V_N(w^*) = V_U.$$

Next, for workers at N firms, the rate of offer arrivals is λ_E. When meeting another N firm, that firm's wage is also drawn from Φ and a job switch occurs if the new wage exceeds the current one. When meeting an R firm, the surplus of the match is $T - V_N(w)$, and the worker gains a fraction α of this surplus:

$$(\rho + \delta)V_N(w) = w + \lambda_E(1 - p_R) \int_w (V_N(w') - V_N(w)) d\Phi(w') + \lambda_E p_R \alpha(T - V_N(w)) + \delta V_U$$

In Appendix B we further characterize the worker's value of being at an R firm and how wages are subsequently set, however this characterization is not necessary for determining p_R or Φ in equilibrium.

Worker Mobility and Steady State The expressions for worker values clarifies the following simple mobility rules in this model. First, in equilibrium all jobs are accepted by workers out of unemployment. Second, R firms in this model will always succeed in "poaching" a worker from N firms due to their ability to bargain, and they will always succeed in retaining workers who receive wage offers from N-firms due to their ability to renegotiate. Third, workers move from one N firm to another only if it results in a higher wage. Finally, since the wage is bid up to the productivity level z when two R firms compete for a worker, it is immaterial which firm retains the worker since it results in zero profit for either firm.

Putting these four mobility rules together implies that for both N and R firms, only meetings with unemployed workers and workers at N firms can result in profitable hires. Thus, in steady state, these fractions are relevant for calculating the relative profitability of each wage-setting mechanism. Balancing flow equations results in a fraction

$$\frac{\delta}{\delta + \lambda_U}$$

of unemployed workers and a fraction

$$\frac{1 - p_R}{1 + \kappa p_R}$$

of those employed are at N firms. Finally, conditional on being at an N firm, the steady state probability that a worker is employed at a wage less than or equal to w is given by

$$G_N(w) = \frac{(1 + \kappa p_R)\Phi(w)}{1 + \kappa p_R + \kappa(1 - p_R)\tilde{\Phi}(w).}$$

Equilibrium Wages at Posting Firms Following Burdett and Mortensen (1998), we allow $\rho \to 0$ which results in the firm's objective to be the maximization of the steady state profit rate given by $l(w)(z - w)$ where $l(w)$ is the measure of workers at the firm in the steady state. We find that steady state profit from a wage offer of w is

$$\Pi_N(w) = \frac{1}{\delta} \frac{z - w}{(1 + \kappa p_R + \kappa(1 - p_R)\tilde{\Phi}(w))^2}.$$

For Φ to be non-degenerate in equilbrium, $\Pi_N(w)$ must be the same for all wages in the support of Φ. In particular, since

$$\Pi_N(w^*) = \frac{1}{\delta} \frac{z - w^*}{(1 + \kappa)^2}$$

we arrive at the following expression for the equilibrium wage posting c.d.f.,

$$\Phi(w) = \frac{1 + \kappa}{\kappa(1 - p_R)} \left(1 - \sqrt{\frac{z - w}{z - w^*}}\right)$$

We will comment below on the relationsip between this expression and its counterpart in Burdett and Mortensen (1998).

Equilibrium Rates of Bargaining To close this simple model we note that for any interior equilibrium ($p_R \in (0, 1)$) it must be the case that the expected profit from acting as an R firm in the market is the same as the profit from posting any wage in the support of Φ as an N firm. In the steady state, the expected profit from meeting a worker as an R firm can be written as:

$$\Pi_R = (1 - \alpha) \left[\frac{1}{1 + \kappa}(T - V_N(w^*)) + \frac{\kappa}{1 + \kappa} \frac{(1 - p_R)}{1 + \kappa p_R} \int_{w^*} (T - V_N(w)) dG_N(w) \right].$$

In words, the probability that the worker is unemployed is $1/(1 + \kappa)$, and in this case the surplus is $T - V_N(w^*)$. With probability $\kappa/(1 + \kappa)$ the worker is employed, and with probability $(1 - p_R)/(1 + \kappa p_R)$ they are employed at an N firm.

The integral in the above expression gives the expected value of the surplus in this case, where G_N is the steady state distribution of workers over wages at N firms. Since Φ is a known function, an analytical expression for Π_R can be derived (for the limiting case of $\rho \to 0$), which is

$$\Pi_R = (1-\alpha)\frac{(z-w^*)}{\delta(1+\kappa)^2}\left[\frac{1+\kappa p_R}{1+\kappa p_R\alpha} + 2\log\left(\frac{1+\kappa-\kappa p_R(1-\alpha)}{1+\kappa p_R\alpha}\right)\right].$$

Proposition 1 below summarizes the characterization of p_R in equilibrium that can be found when imposing the condition $\Pi_N = \Pi_R$ for any equilibrium $p_R \in (0,1)$. Further details are given in Appendix B.

Proposition 1. *Taking $\rho \approx 0$, the equilibrium fraction p_R is characterized by the solution to:*

$$(1-\alpha)\left[\frac{1+\kappa p_R}{1+\kappa p_R\alpha} + 2\log\left(\frac{1+\kappa-\kappa p_R(1-\alpha)}{1+\kappa p_R\alpha}\right)\right] = 1$$

which implies that p_R is strictly increasing in κ and strictly decreasing in α. Furthermore, the inequalities:

$$(1-\alpha)\left[\frac{1+\kappa}{1+\kappa\alpha}\right] \geq 1$$

and

$$(1-\alpha)\left[1+2\log(1+\kappa)\right] \leq 1$$

define the conditions under which $p_R = 1$ and $p_R = 0$, respectively. Finally, if $p_R < 1$, the distribution of posted wages, Φ is:

$$\Phi(w) = \frac{1+\kappa}{\kappa(1-p_R)}\left(1-\sqrt{\frac{z-w}{z-w^*}}\right)$$

This expression for the wage distribution Φ generalizes the case considered in Burdett and Mortensen (1998), and reduces to their expression when $p_R = 0$. It is not defined when $p_R = 1$, since any N firm entering the market at this point only expects to hire unemployed workers (R firms will renegotiate and retain their workers), and hence offers the reservation wage, w^*.

Proposition 1 provides a simple characterization of wage-setting across markets in terms of two key market primitives: α, the fraction of the surplus that the worker obtains through bargaining, and κ, the relative rate of on-the-job employment offers to separations. In particular, for each κ, there exists a lower bound $\underline{\alpha}(\kappa)$ below which all firms choose to bargain and renegotiate, and an upper bound $\overline{\alpha}(\kappa)$ above which all firms choose to post. Figure 2 depicts these thresholds, in addition to the locus of combinations of (α,κ) at which p_R takes various interior values.

The intuition for this result is relatively clear. For α it is obvious that increases in the fraction of the surplus that a worker claims through bargaining will reduce the incentive for a firm to bargain. As κ increases, the average number of offers a worker receives before exogenous separation increases, and the value of being able to renegotiate increases. This basic underlying tension also underlies the extended model developed in the next section, so that it is beneficial to see whether the empirical evidence is consistent with this implication of the model.

2.3 Examining the Model's Predictions

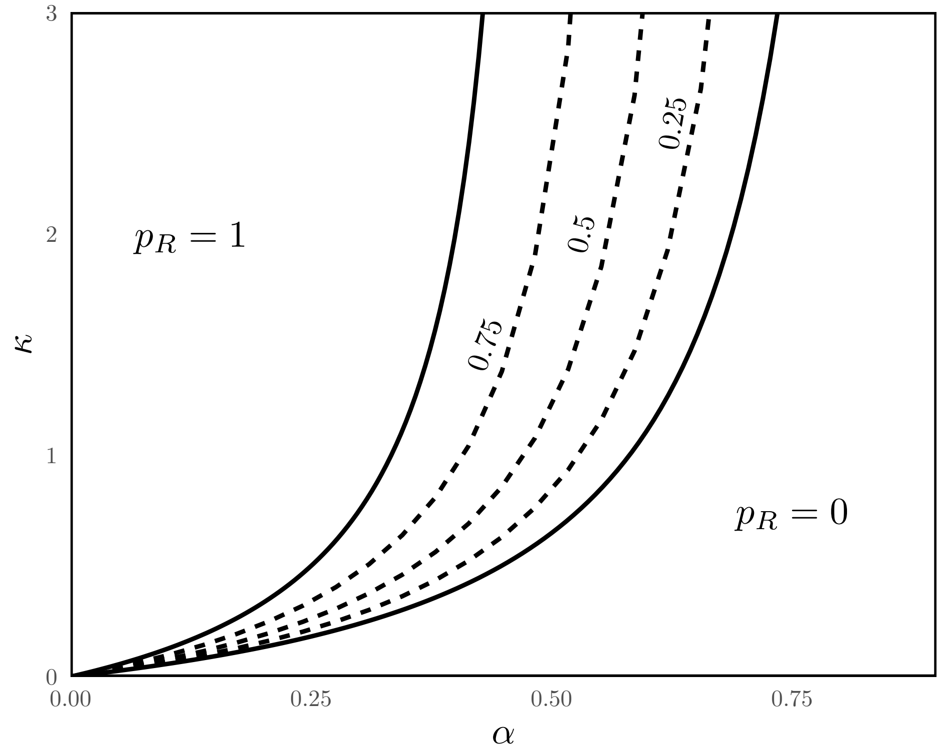
The most important prediction arising from Proposition 1 is that p_R, the fraction of bargaining and renegotiating firms, is increasing in κ. While κ is not directly observable in the data, a reasonable proxy is available, which is the ratio of employer-employer transitions (EE) to separations into unemployment (EU).[9] We define this proxy as k, and measure it

[9] In Appendix B we show that

$$\frac{EE}{EU} = \kappa\left(p_R + \frac{1+\kappa}{\kappa^2}\log\left(\frac{1+\kappa}{1+\kappa p_R}\right) - \frac{1-p_R}{\kappa}\right),$$

an increasing function of κ, which demonstrates the suitability of this proxy.

Figure 2: Equilibrium Space: Homogenous Match Productivity



This figure shows how the equilibrium fraction of negotiating firms, p_R, varies in the space (κ, α) for the case in which the distribution of match productivities is degenerate in a given market. The dashed lines trace out combinations such that $p_R = 0.75$, $p_R = 0.5$, and $p_R = 0.25$. The solid lines trace the threshold values of α such that $p_R = 1$ and $p_R = 0$.

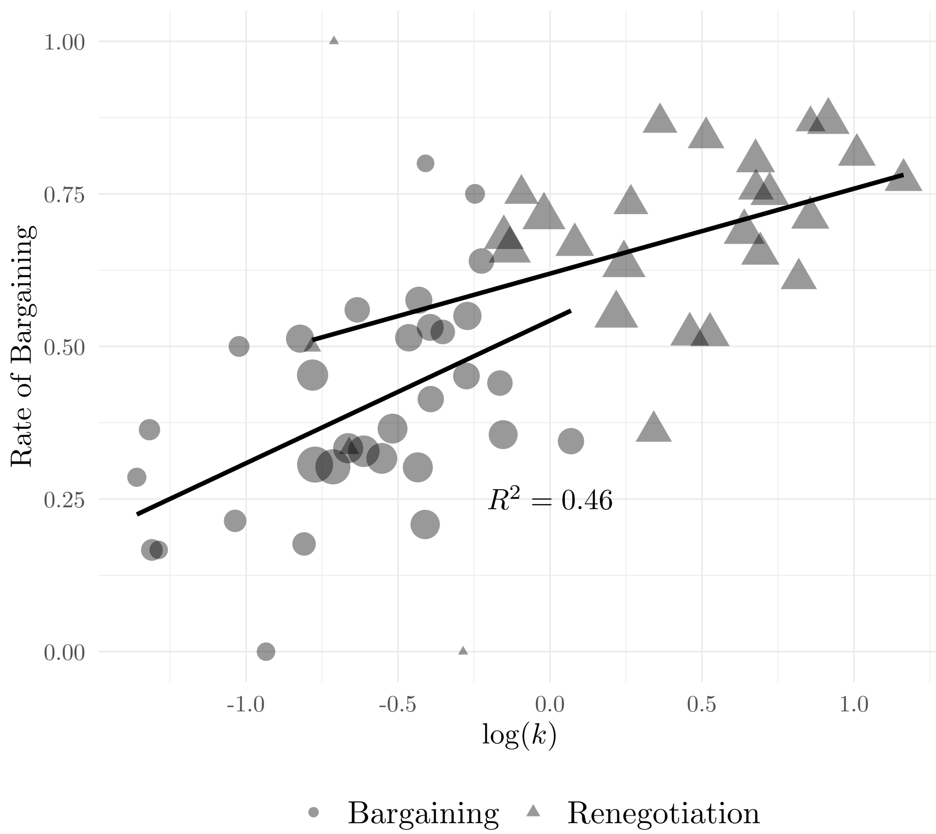
at a monthly frequency across demographic groups in the CPS. We match the proxy to average rates of reported bargaining for those same demographic groups in the Hall and Krueger (2012) data. Since this only tests the model's predictions on bargaining, we go to the Labor Market Module of the *Survey of Economic Expectations* (SCE) and retrieve a measure of renegotiation, which is the fraction of workers in a demographic group that report a positive probability that their current employer would match an outside offer. Figure 3 shows the results, and confirms that the model's predictions are verified across demographic groups. Our proxy for κ, k, predicts bargaining and renegotiation with an overall R^2 of 46 percent.

2.4 Contact Rates in Equilibrium

While we treat the rate parameters $(\lambda_U, \lambda_E, \delta)$ as primitives that are allowed to vary across markets, the contact rates in particular (λ_U and λ_E) can be considered to be equilibrium outcomes determined by differences in z, the productivity associated with each market. These outcomes are determined through the vacancy posting decisions of firms. Let $\tilde{V}_B$ be the measure of vacancies of type j created in equilibrium, where $B \in \{R, N\}$. Each vacancy, no matter of what type, requires a flow cost of $c > 0$ while being held open. The total number of vacancies is $\tilde{V} = \tilde{V}_R + \tilde{V}_N$ and undirected search implies that

$$p_R = \frac{\tilde{V}_R}{\tilde{V}_N + \tilde{V}_R}.$$

Figure 3: Equilibrium Prediction in Data



This figure shows the cross-sectional relationship between k (the ratio of job-to-job transitions to separations) and rates of bargaining and renegotiation across demographic cells. Each point on the graph represents a combination of age category, sex, and education category. Rates of bargaining are taken from survey data collected in Hall and Krueger (2012). Rates of renegotiation are measured as the fraction of individuals in the Survey of Consumer Expectations Labor Market Module who report a positive probability that their employer would match an outside wage offer. Measures of k for each demographic cell are taken from the *Current Population Survey* basic monthly files for 2008 and 2015.

On the worker side, unemployed workers produce one unit of search effort, while employed workers provide $\mu_E < 1$ units of search effort, yielding the relationship

$$\lambda_E = \mu_E \lambda_U.$$

Contact rates are determined by a constant returns to scale matching function, such that:

$$\lambda_U = f(\nu), \qquad \lambda_E = \mu_E f(\nu)$$

where ν is market tightness, the ratio of posted vacancies to total units of search which is given by

$$\nu = \frac{\tilde{V}_R + \tilde{V}_N}{U + \mu_E(1-U)}.$$

Since a firm creating a vacancy is free to choose either type of wage-setting protocol, total vacancies and market tightness are determined by the two free entry conditions

$$(1-p_R)\left[q(\nu)\Pi_N(p_R,\kappa,\alpha) - c\right] = 0, \qquad p_R\left[q(\nu)\Pi_R(p_R,\kappa,\alpha) - c\right] = 0$$

where $q(\nu) = f(\nu)/\nu$ is the Poisson rate at which firms meet workers. When $p_R \in (0, 1)$, both free entry conditions hold, while they hold with complementary slackness when p_R is at a corner.

Now consider an increase in productivity z, which leads to an increase in the profitability of both vacancy types. In equilibrium the market clears with an increase in both types of vacancies, resulting in an increase in λ_E, necessitating that R vacancies must go up proportionally more than N vacancies. Thus, all else equal, bargaining and renegotiation is increasing in the productivity of workers in the labor market (and consequently is related to higher wages and lower rates of unemployment, as in Figure 1).

2.5 Allowing for Alternative Wage Contracts In Equilibrium

While allowing for firms to choose their wage-setting protocol is a feature that we share with very few other papers (Postel-Vinay and Robin, 2004; Doniger, 2015; Michelacci and Suarez, 2006; Cheremukhin and Restrepo-Echavarria, 2021), it is still natural to question whether we have specified the most reasonable choice set for firms. Our model features two particularly stark assumptions. First, the ability to post wages and the ability to renegotiate wages are mutually exclusive and second, firms that post wages are constrained to a fixed wage offer over time. We offer three perspectives that attempt to justify the limited space of contracts that we consider.

First, we consider the two wage-setting protocols in our model to be important benchmarks (potentially the most important) in the labor market search literature both theoretically and empirically. From this perspective we feel that it is most natural to include these two choices in a theoretical study.

Second, we view the choice to post a non-negotiable wage as an act in which the firm ties its hands with respect to wage negotiations, for example by setting up a human resources intermediary that is instructed to post only a single wage. In such a case, it follows that if the firm has credibly committed to not negotiate wages today then it will not be able to renegotiate in the future. On the other hand, although it is clear that N firms could improve their retention by offering more complex wage-tenure contracts, the general insights of the model ought to be identical as long as this retention is imperfect. In this sense we have imposed a limitation for the sake of tractability and insight, which is something we share with many empirical search models that feature wage posting (Bontemps et al., 1999; Engbom and Moser, 2017; Meghir et al., 2015; Shephard, 2017).

Third, for this simple model we are able to theoretically examine equilibria when these restrictions on contracts and renegotiation are relaxed. Technical details for this analysis are provided in Appendix B.5. In the first case we allow the firm to post an optimal wage-tenure contract in the spirit of Stevens (2004), in which the firm allows the worker to buy the right to all profits in the future. In the second case, we consider equilibria in which the firm is able to post a non-negotiable wage and later renegotiate when the worker receives a credible outside option. In both cases we show that including either option drives N firms and R firms out of the market, leading to equilibrium values of $p_R = 0$ and $p_N = 0$. In other words, it is not possible to sustain heterogenous wage contracts in these extensions, and they provide counterfactual outcomes in which all workers report receiving take-it-or-leave-it offers at their current job.

3 The Model with Worker and Match Heterogeneity

In this section, we extend the model to allow for permanent differences in worker productivities as well as differences in match productivity. We first derive simple rules for mobility and bargained wages taking as given the conditional distribution of match productivity at R-firms ($F_\theta(\cdot|R)$) and the wage offer distribution from N-firms (Φ). This setup allows for a general selection rule that nests our equilibrium model while also applying to alternative assumptions on how firms choose wage-setting protocols. Tractability of the model is maintained by identifying a sufficient statistic for worker mobility, and by showing that wages are multiplicative in permanent worker ability. The result is a characterization of steady states and wage formulae that can be flexibly brought to administrative wage data. For example, Caldwell and Harmon (2019) have used our framework to understand the role played by network linkages in determining wages by shaping workers' outside options. The flexibility of our model is crucial here, since wages will exhibit different sensitivities to this outside option across different markets.

In the next stage of the analysis, we show how the wage posting distribution Φ is determined in equilibrium when productivities are idiosyncratic across all worker-firm matches. An interesting implication of the model in this case is that mobility decisions can be inefficient, in that workers may select a job match that is less productive than the other choice currently available to them. The quantitative importance of inefficient mobility will be assessed using the estimated model. Finally, we write the necessary conditions for an internal equilibrium in wage-setting protocols to exist, and numerically verify that the comparative static properties with respect to κ and α derived in the simple model are preserved.

3.1 Environment

Market Primitives Flow output z for a worker-firm pair is given by

$$z = a\theta$$

where a is the worker's permanent and idiosyncratic ability and θ is specific to the match. For now, readers can assume that when a worker meets a firm, θ is drawn from a conditional distribution $F_\theta(\cdot|B)$ where $B \in \{R, N\}$. We will later impose that these conditional distributions are equal for our equilibrium model, but the results here on mobility and wage-setting apply in a more general setting[10]. We maintain our previous assumptions on continuous time and discounting (ρ), the rate of separation of jobs (δ), and contact rates ($\lambda_E < \lambda_U$). We assume that the flow utility of leisure is proportional to ability, giving $b(a) = ba$[11].

Wage-Setting Protocols We maintain the assumption that R firms bargain with workers by awarding a share α of the surplus to the worker, and renegotiating when the worker's outside option improves. We also assume that N firms now offer *piece-rate* wages such that total compensation for a worker is $\omega a\theta$, where ω is the piece rate. We define $w = \omega\theta$ and assume that these offers are drawn from the distribution Φ. In Section 3.5 we will consider how Φ and p_R are determined in equilibrium.

3.2 Worker Values and the Maximum Attainable Wage

As in the simple model, it is instructive to consider two objects: a type a worker's value of being employed at a job of type N with wage w ($V_N(a, w)$), and the total value to the worker and the firm when $B = R$ and match productivity is

[10] In particular, these results apply when firms are characterized by some productivity that is shared across matches and make their protocol choice ex-post, as in Doniger (2015).

[11] As is well-known in models of this type, this assumption guarantees that values and wages are multiplicatively separable in a (Postel-Vinay and Robin, 2002)

θ ($T(\theta)$). Beginning with the latter object, we can write:

$$(\rho+\delta)T(a,\theta) = a\theta + \lambda_E p_R \alpha \int_\theta [T(a,y) - T(a,\theta)]^+ dF_\theta(y|R) + \lambda_E(1-p_R)\int [V_N(a,w) - T(a,\theta)]^+ d\Phi(w) + \delta V_U(a) \quad (1)$$

where $[\cdot]^+$ is defined by the operator $\max\{\cdot, 0\}$. Here, $a\theta$ is the total flow value of the match (z) under perfectly transferable utility between the worker and the firm. At rate λ_E, the worker meets other vacancies, and with probability p_R this vacancy is of type R. All realizations of match productivity at this vacancy, y, in which the wage is renegotiated but no job switch occurs, simply result in a transfer of value between the worker and the firm, resulting in no change to the total value. However when $T(a,y) > T(a,\theta)$, Bertrand competition bids up the worker's outside option to $T(a,\theta)$, and they subsequently negotiate an increase in value equivalent to a fraction α of the surplus, $\alpha(T(a,y) - T(a,\theta))$. Alternatively, with probability $1-p_R$ the vacancy is of type N. As before, all cases in which $V_N(a,w) < T(a,\theta)$ may result in renegotiation only, without affecting the total value. If the wage draw from Φ is sufficiently high, the worker transitions to the new job, generating an increase $V_N(a,w) - T(a,\theta)$ in value. Finally, at rate δ, the job is destroyed, with the worker receiving the value of unemployment. As is standard, we assume that a free entry condition restricts the firms' value of an open vacancy to zero.

The object V_N can be similarly recursively defined as:

$$(\rho+\delta)V_N(a,w) = aw + \lambda_E p_R \alpha \int [T(a,y) - V_N(a,y)]^+ dF_\theta(y|R) + \lambda_E(1-p_R)\int [V_N(a,y) - V_N(a,w)]^+ d\Phi(y) + \delta V_U(a) \quad (2)$$

Since renegotiation of the wage is not possible at this firm, a wage can be successfully negotiated at all N-vacancies with match value y such that $T(a,y) > V_N(a,w)$. According to our assumptions, the worker will receive a fraction α of the surplus $T(a,y) - V_N(a,w)$ generated by this match.

While worker mobility might first appear difficult to characterize in this setting, our first result identifies a sufficient statistic that dictates worker mobility regardless of the type of wage-setting protocol used by the incumbent or the poaching firm. It can be derived by simple application of recursive logic, noting that each of equations (1) and (2) define an identical operator, and can hence be satisfied by a single value function.

Proposition 2. *Let the objects T and V_N be defined as the solutions to equations (1) and (2). Then $T(a,w) = V_N(a,w)$ for all w, meaning that earning a wage w at a posting firm is equivalent to receiving the full value of a match at a negotiating firm with productivity w. Furthermore, T is strictly monotonic in w, implying that a worker switches jobs if and only if the maximum attainable wage at the poaching employer is higher than that attainable at the incumbent employer.*

This proposition identifies the *maximum attainable wage*, defined as the offered wage at an N-job and the match productivity at an R-job, as the statistic that defines whether a worker switches jobs. The maximum attainable wage has a sampling distribution M given by:

$$M(w) = p_R F_\theta(w|R) + (1-p_R)\Phi(w)$$

and it provides two crucial simplifications. First, worker flows are defined in terms of a single state variable, providing convenient analytical characterizations of distributions in steady state. Second, match values can be solved for in terms of a single recursive object, T, providing convenient analytical expressions for negotiated wages. It can now be written as:

$$(\rho+\delta)T(a,\theta) = a\theta + \lambda_E p_R \alpha \int_\theta (T(a,y) - T(a,\theta)) dF_\theta(y|R) + \lambda_E(1-p_R)\int_\theta (T(a,y) - T(a,\theta)) d\Phi(y) + \delta V_U(a)$$

while the value of unemployment can now be defined as:

$$\rho V_U(a) = ba + \lambda_U p_R \alpha \int_{w^*(a)} (T(a,y) - V_U(a)) dF_\theta(y|R) + \lambda_U \int_{w^*(a)} (T(a,y) - V_U(a)) d\Phi(y)$$

where $w^*(a)$ is the reservation value of the maximum attainable wage, defined by $T(a, w^*(a)) = U(a)$. Examining this pair of equations, it is immediate that the system of equations is multiplicatively separable in ability, a, and therefore that:

$$T(a,\theta) = aT(\theta), \qquad V_U(a) = aV_U$$

where T and V_U are recursively defined as the solution to:

$$(\rho+\delta)T(\theta) \;=\; \theta + \lambda_E p_R \alpha \int_\theta (T(y) - T(\theta)) dF_\theta(y|R) + \lambda_E(1-p_R) \int_\theta (T(y) - T(\theta)) d\Phi(y) + \delta V_U \tag{3}$$

$$\rho V_U \;=\; b + \lambda_U p_R \alpha \int_{w^*} (T(y) - V_U) dF_\theta(y|R) + \lambda_U \int_{w^*} (T(y) - V_U) d\Phi(y). \tag{4}$$

Defining the reservation wage as the solution to $T(w^*) = V_U$, it is immediately clear that reservation wages (per ability unit) are constant across all workers in the same market regardless of ability. Notice that worker mobility in this model opens up the possibility of inefficient job-to-job transitions. These will occur whenever a worker is poached from an N firm by a less productive R firm whose productivity is nevertheless greater than the N firm's posted wage. Equivalently, an N firm fails to poach a worker from a less productive R firm. We characterize this inefficiency more carefully in Section 3.5.

3.3 Bargained Wages

Using the simplification that values are multiplicative in a, we now turn to characterizing the wage that is bargained between a worker and an R firm. From here on, we take it as given that all values and wages are written per ability unit. Let $V_R(\theta, w)$ be the value that is afforded to the worker when they are hired from an N firm at wage w. Our assumption on bargaining requires that:

$$V_R(\theta, w) = V_N(w) + \alpha(T(\theta) - V_N(w)).$$

Alternatively, when hiring from an R firm with productivity $w < \theta$, the worker's outside option is bid up to $T(w)$ through Bertrand competition and the bargained wage solves:

$$V_R(\theta, w) = T(w) + \alpha(T(\theta) - T(w))).$$

By Proposition 2, we know that T and V_N are equivalent, and so the maximum attainable wage is also a sufficient statistic for defining the worker's outside option in bargaining. When the worker is hired out of unemployment, the same problem is solved using $T(w^*)$ as the outside option.

Let $\varphi_R(\theta, q)$ be the solution for the wage bargained between an R firm with match productivity θ and a worker with an outside option characterized by the maximum attainable wage q. The value $V_R(\theta, q)$ can be written as:

$$(\rho + \delta + \lambda_E \tilde{M}(\theta)) V_R(\theta, q) = \varphi_R(\theta, q) + \lambda_E p_R (1-\alpha) \int_q^\theta (T(y) - T(q)) dM(y)$$
$$+ \lambda_E p_R \int_\theta [(1-\alpha)T(\theta) + \alpha T(y)] dF_\theta(y|R) + \lambda_E(1-p_R) \int_\theta T(y) d\Phi(y) \tag{5}$$

Equations (3)-(5) can be combined to derive an expression for wages at R jobs, and for the reservation wage. Here we present the solution only, leaving the algebraic details for Appendix C.3. We find that

$$\varphi_R(\theta, q) = \alpha\theta + (1-\alpha)q - \lambda_E p_R (1-\alpha)^2 \int_q^x \frac{\tilde{F}_\theta(y|R)}{\rho + \delta + \lambda_E p_R \alpha \tilde{F}_\theta(y|R) + \lambda_E (1-p_R)\tilde{\tilde{\Phi}}(y)} dy \tag{6}$$

or equivalently:

$$\varphi_R(\theta, q) = \theta - (1-\alpha) \int_q^z \frac{\rho + \delta + \lambda_E p_R \tilde{F}_\theta(y|R) + \lambda_E (1-p_R)\tilde{\tilde{\Phi}}(y)}{\rho + \delta + \lambda_E p_R \alpha \tilde{F}_\theta(y|R) + \lambda_E (1-p_R)\tilde{\tilde{\Phi}}(y)} dy.$$

Similar work leads to a reservation wage equation:

$$w^* = b + (\lambda_U - \lambda_E) \int_{w^*} \frac{\alpha p_R \tilde{F}_\theta(y|R) + (1-p_R)\tilde{\tilde{\Phi}}(y)}{\rho + \delta + \lambda_E \alpha p_R \tilde{F}_\theta(y|R) + \lambda_E (1-p_R)\tilde{\tilde{\Phi}}(y)} dy. \tag{7}$$

Equation (6) conveniently nests the wage-setting formula derived by Cahuc et al. (2006), which can be obtained as a special case when $p_R = 1$. This nesting property is useful because it endows the model with a crucial degree of freedom when using bargaining models of wage setting in empirical applications. In the next section we briefly discuss the additional empirical flexibility that is provided by the model and its potential usefulness.

3.4 Implications for Frictional Wage Dispersion

Since the effective wage is scaled by each worker's ability, a, the model preserves a wage decomposition that is typical in empirical search: log wages can be decomposed into a component that is attributable to worker ability ($\log(a)$) and a residual component ε_{it} that is a function of the worker's history of employment and job offers. For a worker i at time t, we have

$$\log(w_{it}) = \log(a_i) + \varepsilon_{it}.$$

The residual ε_{it} is often referred to as *frictional* or *residual wage dispersion* since it is attributable only to differences in randomness in undirected search. In models of wage posting[12] ($p_R = 0$), ε_{it} is a function only of the match productivity (θ_{it}) at the current firm, while in models of bargaining and renegotiation ($p_R = 1$), ε_{it} depends both on the current value of match productivity and the outside option most recently used to bargain the wage (q_{it}). In our hybrid model, either of the above could be true, depending on whether or not worker i is currently at an R-firm. Hence, the probability that ε_{it} depends on outside options is determined by the market-level primitive p_R. Furthermore, equation (6) demonstrates that the strength of this dependence is determined both by α and p_R in equilibrium.

Recent empirical research using matched employer-employee data has shown that the effect on wages of origin firms (Di Addario et al., 2020) and of outside options (Caldwell and Harmon, 2019) varies substantially across labor markets and across workers. Our theoretical framework is able to rationalize this variation with differences in the fraction of bargaining firms (p_R) across markets. Notice that p_R affects wage dynamics through two channels. Most directly, a fraction $1 - p_R$ of all offered wages are posted and non-negotiable, implying that individual worker's outside options have no influence on the realized wage. Indirectly, the wage equation (6) shows that the fraction p_R of negotiating firms in the market influences the contribution of the outside option to the bargained wage. Since most empirical work using matched data exploits the log-additive model of Abowd et al. (1999), in Appendix C.3.1 we show how the model can be adjusted to produce a wage equation in which log wages are comprised of a firm and worker fixed effect in addition to a residual that depends on the type of firm and (potentially) the outside option. This wage equation allows a structural interpretation of

[12] Notable examples are Bontemps et al. (1999), van den Berg and Ridder (1998), Engbom and Moser (2017)

wage decompositions that use worker and firm fixed effects, adopting the approach of Bagger et al. (2014) to our setting with heterogeneous wage-setting. Equation (6) also demonstrates that, along with α, p_R is crucial in determining the "steepness" of the wage ladder with job tenure at R-firms, and therefore is highly influential in shaping wage dispersion due to search frictions.

3.5 Wage-Setting Choices in Equilibrium

In the previous sections we treated the key objects p_R, $F_\theta(\cdot|R)$ and Φ as fixed, solving for worker mobility and bargained wages accordingly. We now consider how these objects are determined in equilibrium under different sets of assumptions on market primitives. In Section 2 we saw that equilibrium rates of bargaining are increasing in κ and decreasing in α, a dynamic that will be preserved in this extended model. We begin by solving for the wage-posting choices of N firms, using the assumption that match productivity θ is identically and independently drawn across matches at the time of meeting a worker, and *after* the choice of the bargaining protocol for the vacancy. N-firms are able to post a wage schedule φ that sets a per ability unit wage for each productivity, so a worker of ability a at a firm with productivity θ will receive a wage $a\varphi(\theta)$.[13]

To solve for φ in the steady state, consider the profit maximization problem for a firm with match productivity θ, fixing the equilibrium offer distribution Φ. Once again, the sampling distribution of maximum attainable wages can be defined as $M(x) = p_R F_\theta(x) + (1 - p_R)\Phi(x)$, and so given a wage offer w, the rate at which the worker transitions to another firm is $\lambda_E \tilde{M}(w)$. Given this, the value to the firm of a match at wage w is

$$\frac{\theta - w}{\rho + \delta + \lambda_E \tilde{M}(w)}.$$

Since the wage is posted and non-negotiable, the firm must factor in the probability that a wage offer w is acceptable to the worker, which in turn depends on the worker's maximum attainable wage (set equal to w^* for the unemployed). In Appendix C.1 we characterize the distribution of workers over wages and employment states, and in particular we find that the fraction

$$\frac{1}{1 + \kappa \tilde{M}(w)}$$

will find a wage offer of w preferable to their current employment. Thus, the profit to the firm of a wage offer of w is:

$$\Pi_N(\theta, w) = \frac{1}{1 + \kappa \tilde{M}(w)} \frac{\theta - w}{\rho + \delta + \lambda_E \tilde{M}(w)}.$$

As is typical in wage-posting models, the firm faces a trade-off between the size of flow profits on the job and the rate of hiring and retention. Assuming that φ is monotonically increasing in θ, the equilibrium offer distribution is given by

$$\Phi(x) = F_\theta(\varphi^{-1}(x))$$

and the equilibrium φ is defined as a fixed point: φ implies Φ, which maps to an optimal wage for each θ, resulting in the wage schedule φ. In Proposition 3, we verify that the technical conditions are met in order to characterize the equilibrium wage offer function (almost everywhere) as a first order differential equation.

Proposition 3. *Assume that match productivities are drawn from a continuous distribution F_θ and that the decision to bargain is made* ex-ante *(before the realization of productivity). Then posting (N) firms' optimal wage offer strategies are*

[13] We will see that φ is a deterministic mapping from match productivity to wage offer that determines Φ, the wage offer distribution in equlibrium.

given by a deterministic function φ that is (1) monotonically increasing; (2) lower semi-continuous; (3) almost everywhere differentiable; and (4) satisfies $\varphi(w^*) = w^*$. The distribution of posted wages, Φ is therefore given as:

$$\Phi(w) = F_\theta(\varphi^{-1}(w))$$

Proof. The proof is given as a combination of Lemmas 1-7 in Appendix C.2. ∎

Rearranging the firm's first order conditions for their choice of posted wage results in a first order differential equation that can be written as:

$$\varphi'(x) = \frac{(1-p_R)f_\theta(x)}{\left[\lambda_E(\theta-\varphi(\theta))\left(\frac{1}{\rho+\delta+\lambda_E p_R\tilde{F}_\theta(\varphi(x))+\lambda_E(1-p_R)\tilde{F}_\theta(x)}+\frac{1}{\delta+\lambda_E p_R\tilde{F}_\theta(\varphi(x))+\lambda_E(1-p_R)\tilde{F}_\theta(x)}\right)\right]^{-1} - p_R f_\theta(\varphi(x))}.$$

This characterization has at least two interpretable features. First, the slope at match productivity x increases proportionally with the density of match productivities at x as well as the fraction of posting firms, $1-p_R$. In other words, the wage offer function gets steeper when there are more posting firms, both globally $(1-p_R)$ and locally $(f_\theta(x))$ to compete with. Second, as productivity θ approaches the reservation value (w^*), the derivative of the wage offer function approaches zero. In other words, the wage offer function tends to be very flat for low realizations of productivity. We will see later on that in equilibrium this creates a sharp spike in the density of wages at N firms near the bottom of the distribution and contributes to the fact that R firms, on average, pay higher wages. The differential equation above is derived from the first order conditions for the firm's wage offer problem, which is necessary only for interior equilibria and not sufficient. Therefore, in solving for φ we must adjust the standard solution algorithm for first order differential equations to globally check the firm's maximization problem at a subset of points. Appendix C.2 provides additional details.

Since match productivity is realized after the choice of bargaining protocol, the expected value of meeting a worker as an N-type vacancy is:

$$\Pi_N(w) = \int_{w^*} \frac{1}{\delta+\lambda_E\tilde{M}(\varphi(\theta))}\frac{\theta-\varphi(\theta)}{\rho+\delta+\lambda_E\tilde{M}(\varphi(\theta))}dF_\theta(\theta)$$

which simplifies (via an application of integration by parts and the envelope theorem) to:

$$\Pi_N(p_R, w^*, \lambda_E, \delta, F_\theta) = \int_{w^*} \frac{\tilde{F}_\theta(y)}{(\rho+\delta+\lambda_E\tilde{M}(\varphi(y)))(\delta+\lambda_E\tilde{M}(\varphi(y)))}dy$$

Details for this derivation can be found in Appendix C.2. Similarly, profit for R firms can be written as:

$$\Pi_R(p_R, \alpha, w^*, \lambda_E, \delta, F_\theta) = (1-\alpha)\int_{w^*} \frac{\tilde{F}_\theta(y)}{(\rho+\delta+\lambda_E\alpha p_R\tilde{F}_\theta(y)+\lambda_E(1-p_R)\tilde{F}_\theta(\varphi^{-1}(y)))(\delta+\lambda_E\tilde{M}(y))}dy$$

An interior equilibrium $p_R \in (0,1)$ can only exist if firms are indifferent between either wage-setting protocol, yielding the equilibrium condition:

$$\Pi_N(p_R, w^*, \lambda_E, \delta, F_\theta, \alpha) = \Pi_R(p_R, \alpha, w^*, \lambda_E, \delta, F_\theta, \alpha). \tag{8}$$

An equilibrium at $p_R = 0$ is sustained if

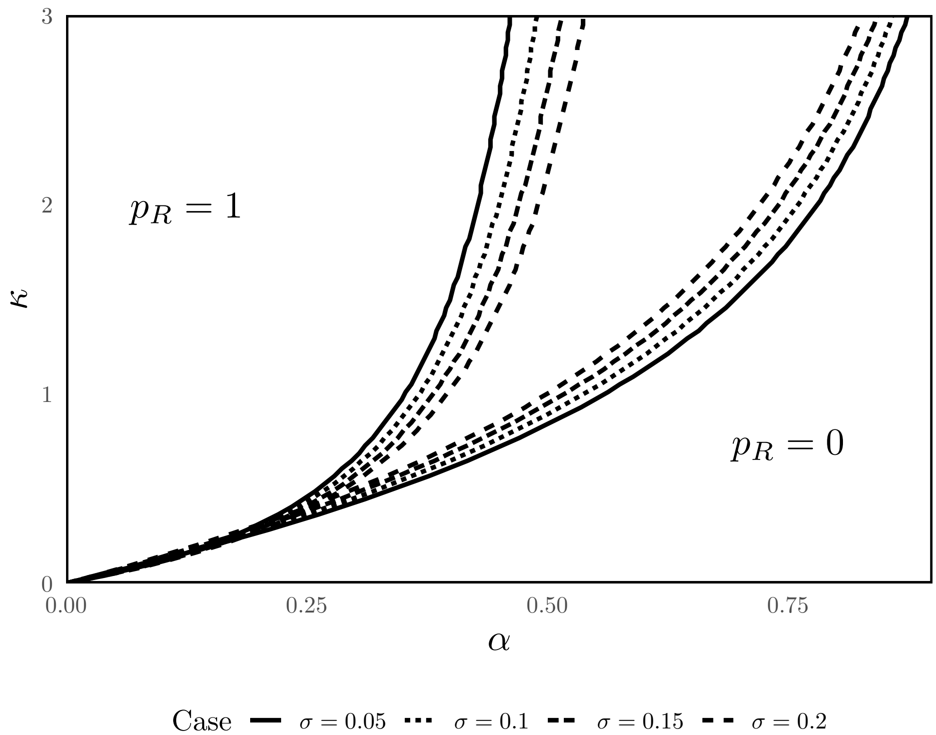
$$\Pi_N(0, w^*, \lambda_E, \delta, F_\theta, \alpha) \geq \Pi_R(0, \alpha, w^*, \lambda_E, \delta, F_\theta, \alpha)$$

and likewise at $p_R = 1$ if

$$\Pi_N(1, w^*, \lambda_E, \delta, F_\theta, \alpha) \leq \Pi_R(1, \alpha, w^*, \lambda_E, \delta, F_\theta, \alpha).$$

While we cannot guarantee that equilibria are unique in this setting, we will see that the realized equilibria are identified directly from the data which mitigates the potential problem of multiple equilibria.

Figure 4: Equilibrium Space: Uniform Productivity Distribution



This figure shows how the threshold values of α and κ between which equilibrium values of $p_R \in (0, 1)$ are possible. These contours are drawn for the case in which F_θ is uniform on the support $[1 - \sigma, 1 + \sigma]$ and wage-setting is chosen *ex-ante*. The figure shows the space in which interior equilibria are possible shrinking as σ grows larger.

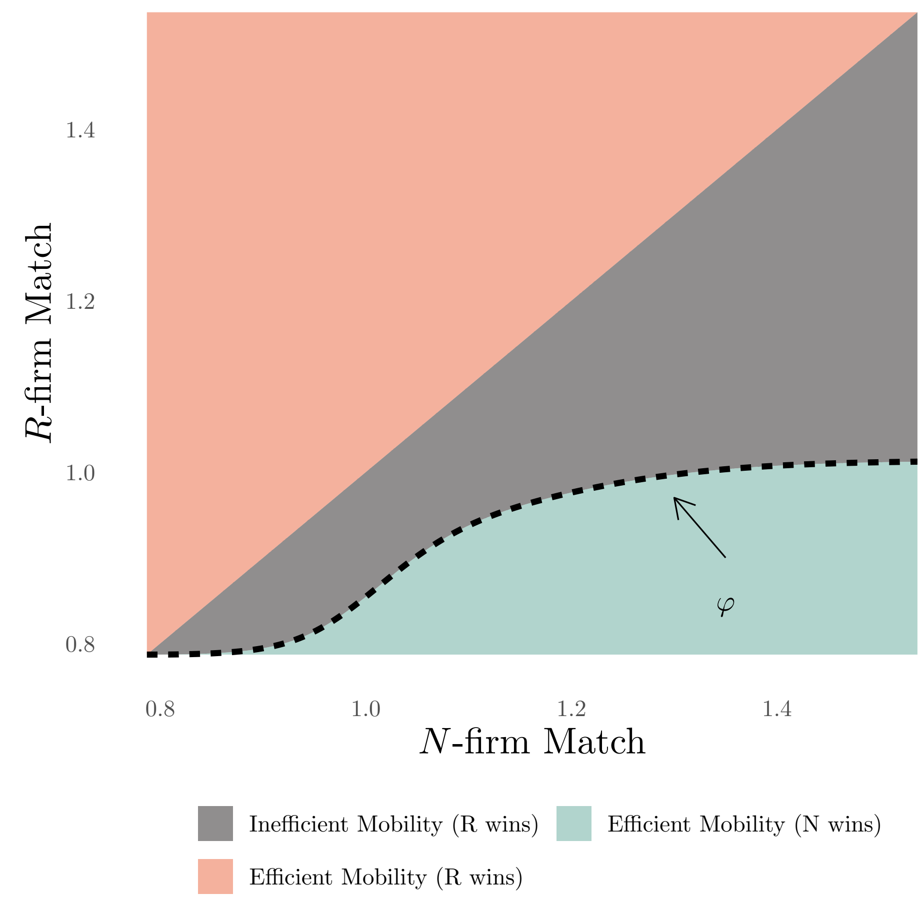
We conclude this section with a numerical exercise to verify that equilibria in this model exhibit similar comparative statics to the model without heterogeneity in productivity. To do this, we once again apply the approximation that $\rho \approx 0$. In this case the expressions for firm profit and reservation wages simplify to dependence on the primitive triple $(F_\theta, \alpha, \kappa)$. Accordingly, the equilibrium p_R in this case depends solely on the triple $(F_\theta, \alpha, \kappa)$. Figure 4 compares the equilibrium space depicted for the homogenous productivity case to cases in which F_θ is a uniform distribution on the interval $[1 - \sigma, 1 + \sigma]$. This figure shows that the general shape of the equilibrium space is preserved, while the space in which interior equilibria are possible shrinks as σ increases.

3.6 Inefficient Mobility

Having characterized the equilibrium wage offer function, φ, we can now more carefully characterize the set of job acceptance and rejection decisions that are bilaterally and socially inefficient. We refer to this phenomenon as inefficient mobility. When a worker is at an R firm with productivity θ, this occurs whenever they meet an N firm and the productivity draw θ' satisfies $\varphi(\theta') < \theta < \theta'$. In this scenario, the incumbent R firm can profitably renegotiate their wage and retain the worker. This occurs at the rate $\lambda_E(1 - p_R)(F_\theta(\varphi^{-1}(\theta)) - F_\theta(\theta))$. Similarly, when workers are at an N firm with productivity θ, job offers from R firms are inefficiently accepted whenever the productivity draw satisfies

$\varphi(\theta) < \theta' < \theta$, which occurs at a rate $\lambda_E p_R(F_\theta(\theta) - F_\theta(\varphi(\theta))$. Figure 5 illustrates the space of encounters and highlights in grey those which result in inefficient job acceptance or rejection decisions.

Figure 5: Inefficient Mobility



This figure shows the combinations of matches at R and N firms that result in efficient and inefficient mobility. An N-firm with match x wins if and only if the wage offer $\varphi(x)$ is greater than the R-firm's match y. When $\varphi(x) < y < x$, the model exhibits inefficient mobility.

3.7 Contact Rates in Equilibrium

We maintain the assumptions from Section 2.4 and allow contact rates to be determined in equilibrium by firms' posting of vacancies subject to a free entry condition. The only modification is that firm profits depend additionally on F_θ, the distribution of match productivities, and the average ability, a, of workers in the market. We therefore rewrite the set of free entry conditions as:

$$q(\nu)\mathbb{E}[a]\Pi_N(p_R, w^*, \lambda_E, \delta, F_\theta, \alpha) \leq c \quad (9)$$

$$q(\nu)\mathbb{E}[a]\Pi_R(p_R, w^*, \lambda_E, \delta, F_\theta, \alpha) \leq c \quad (10)$$

$$p_R = \frac{\tilde{V}_R}{\tilde{V}_N + \tilde{V}_R}, \quad (11)$$

where ν is market tightness, $q(\nu)$ is the rate at which posted vacancies encounter workers, and $\tilde{V}_j$ is the measure of posted vacancies of type j, $j = R, N$. We specify a Cobb-Douglas matching function with elasticity parameter η, giving contact rates:

$$\lambda_U = \nu^\eta, \qquad \lambda_E = \mu_E \nu^\eta, \qquad q = \nu^{\gamma-1},$$

where $\mu_E \in (0, 1)$ is the efficiency of search when employed relative to when unemployed.

4 Identification and Estimation

4.1 Identification

In order to motivate our estimation strategy, we first consider identification of the parameters of the model which are:

$$\beta = \{F_\theta, b, \alpha, \delta, \lambda_E, \lambda_U\}.$$

We do not estimate the discount rate ρ, which we set to 0.005 (monthly frequency). In addition to these parameters, there are two key equilibrium objects, p_R and w^*, that determine wages and mobility outcomes. As we will see, there is a one-to-one relationship between the reservation wage and b, and a one-to-one relationship between p_R and α.

Fixing the probability that an acceptable offer is received, $\tilde{F}_\theta(w^*)$, δ is identified by the flow rate from employment to unemployment (UE), and λ_U is identified by the steady state unemployment rate (U):

$$EU = \delta, \qquad U = \frac{\delta}{\lambda_U \tilde{F}_\theta(w^*) + \delta}.$$

Given knowledge of these parameters, λ_E is identified by the rate at which workers make job-to-job transitions in steady state (EE). This is given by:

$$EE = \lambda_E \int_{w^*} G(x) dM(x)$$

where G is the steady state distribution of workers over maximum attainable wages and M is the sampling distribution of maximum attainable wages. A change of variables gives:

$$EE = \frac{\lambda_E}{\tilde{F}_\theta(x^*)\kappa^2} \left[(1 + \kappa\tilde{F}_\theta(w^*)) \log(1 + \kappa\tilde{F}_\theta(w^*)) - \kappa\tilde{F}_\theta(w^*)\right] \quad (12)$$

where $\kappa = \lambda_E/\delta$. Hence, the triple $(\delta, \lambda_E, \lambda_U)$ is identified up to $\tilde{F}_\theta(w^*)$ using these moments on labor market flows.

Fixing the pair (w^*, F_θ), the equilibrium fraction of R jobs, p_R, can be identified by the steady state fraction of workers at R firms (defined as B) which can be calculated as:

$$B = p_R \int_{w^*} \frac{(1 + \kappa\tilde{F}_\theta(w^*)) f_\theta(x)}{\tilde{F}_\theta(w^*)(1 + \kappa\tilde{M}(x))^2} dx$$

where the integrand is the steady state density of workers at R firms with productivity x (derived in Appendix C.1). Notice that M includes the equilibrium distribution of posted wages which depends on F_θ, w^*, and the previously identified triple of flow rates. We will measure B for each demographic group directly in the survey data of Hall and Krueger (2012). Applying the equilibrium condition in equation (8) identifies the value of α at which firms' profits are equal for both bargaining strategies. Thus, α is also identified by B using this equilibrium constraint.

Finally, we turn to the pair (w^*, F_θ), which we identify by combining data on wages from newly accepted jobs with wages in the steady state. Let W_{SS} be the random variable defined by sampling from the steady state distribution of wages. Similarly, let W_{UE} be defined by random sampling of accepted wages out of unemployment. Since wages can be written in per ability units, we can decompose the first and second moments of both random variables into:

$$\mathbb{E}[\log(W_s)] = \mathbb{E}[\log(a)] + \mathbb{E}[\log(\varepsilon_s)], \qquad s \in \{SS, UE\}$$

and

$$\mathbb{V}[\log(W_s)] = \mathbb{V}[\log(a)] + \mathbb{V}[\log(\varepsilon_s)], \qquad s \in \{SS, UE\}.$$

where ε is the term we introduced in Section 3.4 as frictional wage dispersion. We combine these moments to difference out the contribution of unobserved ability to wages, resulting in

$$\mathbb{E}[\log(W_{SS})] - \mathbb{E}[\log(W_{UE})] = \mathbb{E}[\log(\varepsilon_{SS})] - \mathbb{E}[\log(\varepsilon_{UE})]$$

and

$$\mathbb{V}[\log(W_{SS})] - \mathbb{V}[\log(W_{UE})] = \mathbb{V}[\log(\varepsilon_{SS})] - \mathbb{V}[\log(\varepsilon_{UE})].$$

These moments provide information about wages in equilibrium without any "contamination" from the unknown distribution of individual ability. In order for these two moment conditions to be sufficient, we impose the parametric restriction that F_θ is log-normal with mean zero [14] and standard deviation σ. [15]

We note that the flow value of unemployment b is identified by the reservation wage equation (7). In this way, we are able to identify (and subsequently estimate) the model without specifying the distribution of log ability. As such, when we use the model to perform and estimate counterfactuals, calculations from the model only pertain to the contribution of ε and not a. By contrast, calculations of equivalent statistics in the data contain contributions from both ability and residual wage dispersion.

4.2 Estimation Procedure

In Section 2.3 we validated the simple model's key prediction that the ratio of EE to EU transitions should predict bargaining across markets. We did this by calculating the ratio for separate demographic groups in the Current Population Survey and linking them to reported rates of bargaining and renegotiation for these same demographic groups using data from Hall and Krueger (2012) and the Survey of Consumer Expectations (additional details on data construction can be found in Appendix A). To estimate the model we follow this same strategy, treating each demographic group x as a segmented labor market, and estimating parameters separately for each market. Our identification argument motivates this estimation procedure, in which we impose the identifying conditions that directly identify particular parameters via a minimum distance criterion.

Letting x index demographic groups, we categorize individuals by birth cohort (as determined by their age in 2008, the year of survey), gender, and education. The full set of demographic groups is the product set $\mathcal{X} = \text{Educ} \times \text{Age} \times \text{Sex}$

[14] This is without loss of generality, since the model can be estimated without specifying a location for log productivity.

[15] In principle, instead of using moments one could take the ratio of the characteristic functions $\chi_{W,ue}(t)/\chi_{W,ss}(t)$ for any chosen value of t and identify specifications of F_θ that require more parameters. Since a is independent of ε we get $\chi_{W,s}(t) = \chi_a(t)\chi_{\varepsilon,s}(t)$ for $s \in \{ee, ue\}$.

where education groups are those with (1) less than High School; (2) High School or equivalent; (3) a Bachelor's degree; and (4) more than a Bachelor's degree. The age groups are (1) 20-29; (2) 30-39; (3) 40-49; and (4) 50-59. For the sake of obtaining adequate precision in the estimates, we impose the restriction that b (the flow value of unemployment) and σ (the standard deviation of log match productivity) are common across demographic groups that share an education level.

In order to describe the minimum distance estimation procedure, we partition parameters into two sets:

$$\boldsymbol{\beta}_1 = \{\boldsymbol{\sigma}, \mathbf{b}, \mathbf{w}^*, \mathbf{p_R}\}, \qquad \boldsymbol{\beta}_2 = \{\boldsymbol{\alpha}, \boldsymbol{\delta}, \boldsymbol{\lambda}_\mathbf{E}, \boldsymbol{\lambda}_\mathbf{U}\}$$

where

$$\boldsymbol{\sigma} = \{\sigma(e)\}_{e\in\text{Educ}}, \qquad \mathbf{b} = \{b(e)\}_{e\in\text{Educ}}$$

are parameters specific to each education group and the remaining vectors contain parameters specific to each demographic group:

$$\boldsymbol{\gamma} = \{\gamma(x)\}_{x\in\mathcal{X}}, \gamma \in \{w^*, p_R, \alpha, \delta, \lambda_E, \lambda_U\}.$$

Similarly, we partition moments from the data into:

$$\hat{m}_1(x) = \begin{bmatrix} \hat{B}(x) \\ \widehat{\mathbb{E}}[\log(W_{SS})|x] - \widehat{\mathbb{E}}[\log(W_{UE})|x] \\ \widehat{\mathbb{V}}[\log(W_{SS})|x] - \widehat{\mathbb{V}}[\log(W_{UE})|x] \end{bmatrix}, \qquad \hat{m}_2(x) = \begin{bmatrix} \widehat{EU}(x) \\ \widehat{EE}(x) \\ \hat{U}(x) \end{bmatrix}$$

where $\widehat{\mathbb{E}}$ and $\widehat{\mathbb{V}}$ denote the sample mean and variance, $\hat{B}(x)$ is the fraction of workers of demographic type x in the survey data of Hall and Krueger (2012) who report bargaining, $\widehat{EU}(x)$ is the monthly fraction of type x individuals who transition from employment to unemployment, $\widehat{EE}(x)$ is the monthly rate of type x worker transitions between firms, and $\hat{U}(x)$ is the unemployment rate of type x individuals. These latter three moments are calculated using the CPS basic monthly files in 2008. The wage moments are calculated using the subset of CPS observations that appear in the outgoing rotation group (ORG) files (in which information on earnings is available).

Our minimum distance criterion is written using a nested procedure. After fixing the parameters $\beta_1(x)$, the parameters $\beta_2(x)$ are chosen such that the model exactly fits the moments $\hat{m}_2(x)$ and such that the equilibrium condition for wage-setting (8) holds exactly. This defines a mapping $\hat{\beta}_2(\beta_1, \hat{m}_2)$ given by the solution to the system of equations:

$$\psi_2(\beta_1, \hat{\beta}_2(\beta_1, m_2), m_2) = \begin{bmatrix} \delta - EU \\ \frac{\delta}{\delta + \lambda_U \tilde{F}_\theta(w^*)} - U \\ \lambda_E \frac{1}{\tilde{F}_\theta(w^*)\kappa^2}\left[(1+\kappa\tilde{F}_\theta(w^*))\log(1+\kappa\tilde{F}_\theta(w^*)) - \kappa\tilde{F}_\theta(w^*)\right] - EE \\ \Pi_R(p_R, \alpha, \lambda_E, \lambda_U, \delta, \sigma, w^*) - \Pi_N(p_R, \lambda_E, \lambda_U, \delta, w^*) \end{bmatrix} = 0$$

Since it is triangular, this system of equations can be solved sequentially by first solving for δ, then λ_U, then λ_E, and finally α. As we argued above, w^* and σ are identified by properties of the wage distribution in the steady state (W_{SS}) and for newly formed matches (W_{UE}), while $p_R(x)$ must be chosen such that the fraction of workers at R firms in the steady state is equal to $\hat{B}(x)$. Since we impose the restriction that b is shared across groups x with the same level of education, we add the reservation wage equation to the set of restrictions imposed in the outer estimation loop. We define this set of restrictions as:

$$\psi_1(\beta_1, \beta_2, \hat{m}_1) = \begin{bmatrix} w^* - b - (\lambda_U - \lambda_E)\int_{w^*} \frac{\alpha p_R \tilde{F}_\theta(s) + (1-p_R)\tilde{\Phi}(s)}{\rho + \delta + \lambda_E \alpha p_R \tilde{F}_\theta(s) + \lambda_E(1-p_R)\tilde{\Phi}(s)} ds \\ m_1(\beta_1, \beta_2) - \hat{m}_1 \end{bmatrix} = 0$$

where $m_1(\beta_1, \beta_2)$ is a vector of population moments calculated at the solution of the model. Using these definitions, the full minimum distance estimator is defined as

$$\begin{aligned}\widehat{\boldsymbol{\beta}}_1 &= \arg\min \sum_x \psi_1(\beta_1(x), \hat{\beta}_2(\beta_1(x), \hat{m}_2(x)))'\mathbf{W}(x)\psi_1(\beta_1(x), \hat{\beta}_2(\beta_1(x), \hat{m}_2(x))) \\ \widehat{\boldsymbol{\beta}}_2 &= \{\hat{\beta}_2(\hat{\beta}_1(x), \hat{m}_2(x))\}_{x\in\mathcal{X}}\end{aligned}$$

where each $\mathbf{W}(x)$ is a symmetric, positive semi-definite weighting matrix. Since parameters are shared only within education categories, this minimization can be performed separately for each education category. We calculate standard errors on all parameters and statistics from the estimated model by drawing 100 bootstrap samples of the moments and re-estimating the model for each draw. For each $\mathbf{W}(x)$ we use a diagonal matrix with values of 1 in the first two entries (corresponding to bargaining and the reservation wage equation) and $0.01\times$ the inverse of the moment variance for the remaining entries (corresponding to the wage moments).

4.2.1 Estimating Matching Function Parameters

We conclude this section by considering the estimation of the parameters that determine contact rates through vacancy posting. We need estimates and/or choices of these parameters to enable us to calculate counterfactuals in which contact rates are determined in equilibrium. It is obvious that for each market we can estimate the relative efficiency of search in the employment state by $\hat{\mu}_E = \hat{\lambda}_E/\hat{\lambda}_U$. Typically, η is identified using joint variation in vacancies and contact rates which we do not observe here. For our equilibrium counterfactuals we set $\eta = 0.5$, which is a widely accepted mid-range value (Petrongolo and Pissarides, 2001). Given this, we calculate equilibrium market tightness as $\hat{\nu} = \hat{\lambda}_U^{1/\eta}$. Next, since we observe markets in which $p_R \in (0, 1)$, the inequalities in equations (9) and (10) both hold with equality. We invert equation (9) to get an estimate of the ratio between vacancy costs and average worker ability in the market, $\widehat{c/\mathbb{E}[a]}$, which is sufficient for evaluating the equilibrium condition when calculating counterfactuals.

4.3 Estimates and Model Fit

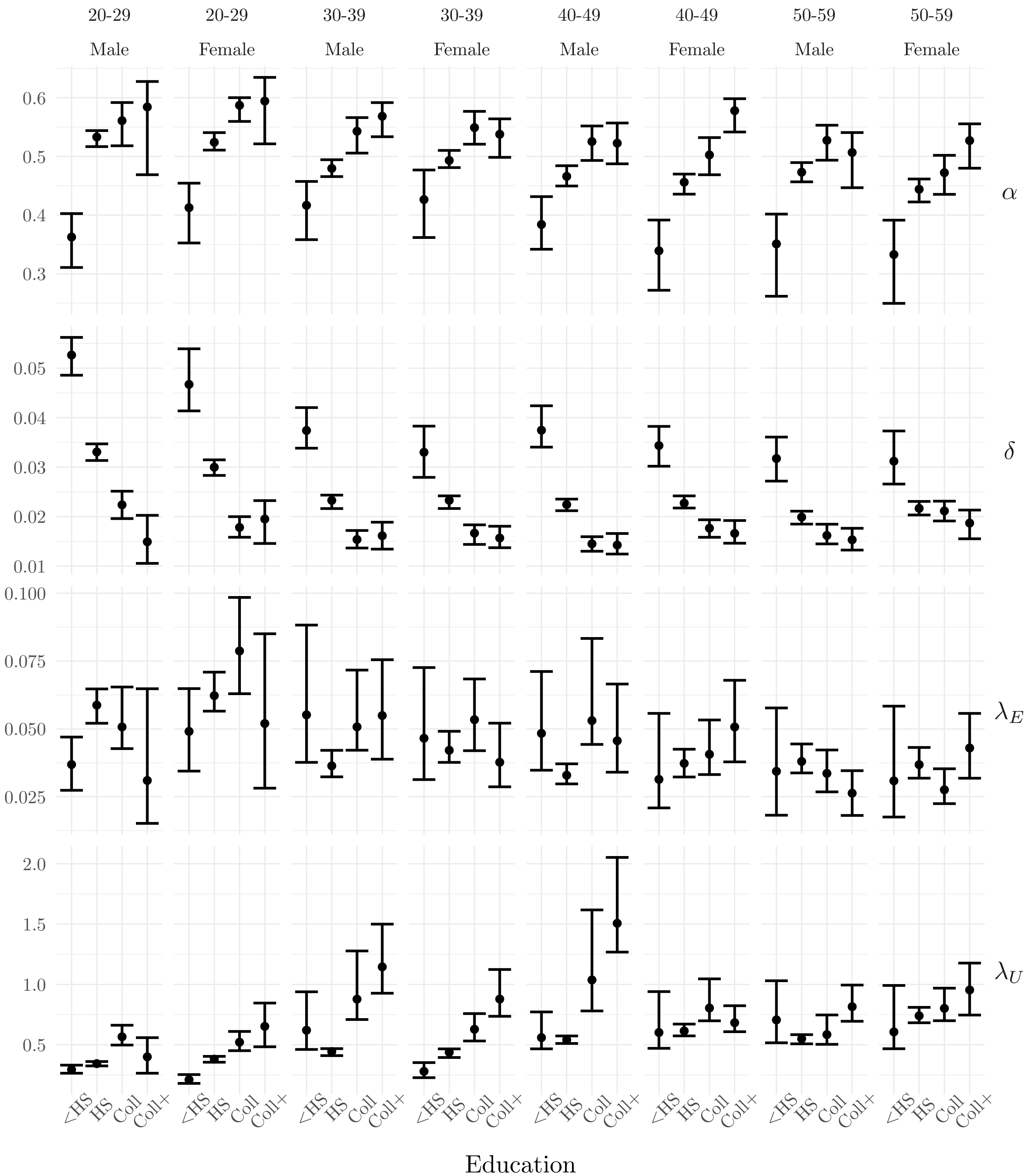
Table 1 reports the estimated values of b and σ, which are shared by all groups within the same education category. We find that b, which governs flow utility in unemployment[16], is decreasing in education. Dispersion in match productivity (σ) is also weakly decreasing in education, but these differences across education groups are not statistically significant.

Due to the large number of group-specific parameters, Figure 6 displays the estimates of our group-specific parameters — α, δ, λ_E, and λ_U — in graphical format with 95 percent confidence intervals. Exact values for these estimates can be found in Table 7. In terms of overall patterns in these parameter estimates, the strongest differences appear between education groups rather than between age or gender categories. Of particular interest is the finding that worker bargaining power, α, is increasing in education and stable between gender categories. Recall that our theoretical analysis identified both α and κ as potential drivers of bargaining and negotiation across markets. Given our identification and estimation strategy, there was a risk that our estimates would rationalize differences in bargaining by assigning lower values of α to men and higher educated workers. Instead, we find that these patterns offset the forces provided by group differences in κ rather than supplementing them.

As we outlined when describing the nested model-fitting procedure in the previous section, we fit separation rates, unemployment rates, and job-to-job transition rates with exact precision. These moments are presented in Table 5 and

[16]Recall that flow utility is ba where a is the worker's permanent productivity.

Figure 6: Estimates of Group-Specific Parameters



This figure graphically shows the estimates of $(\alpha, \delta, \lambda_E, \lambda_U)$ for each combination of age, sex, and education groups. Error bars show 95% confidence intervals for each parameter, obtained using bootstrapped standard errors from 100 simulated samples.

Table 1: Estimates of Education-Specific Parameters

Education	b	σ
<High School	0.26	0.27
	(0.13)	(0.05)
High School	-0.77	0.25
	(0.03)	(0.01)
Bachelor's	-2.09	0.24
	(0.24)	(0.02)
>Bachelor's	-2.05	0.17
	(0.14)	(0.02)

This table shows estimates of b, the flow value of unemployment, and σ, the standard deviation of the log of match productivity. These parameters are shared by all individuals in the same education category. Brackets indicate standard errors, calculated using 100 bootstrap samples.

have by definition perfect model fit. In the outer loop we maximized the fit of rates of bargaining in the steady state, the reservation wage equation, and our two wage moments for each demographic group. We present the moments and the model fit in Table 6. The fit of the reservation wage equation and bargaining moments is almost perfect, which is not surprising given the parameters $p_R(x)$ and $w^*(x)$ are free for each group x in the minimization routine. By contrast, we have only 8 parameters[17] that can be used to fit the 64 wage moments across groups. Overall, the fit of these moments is also quite good taking into consideration the standard errors of the sample moments.

5 Analysis of the Estimated Model

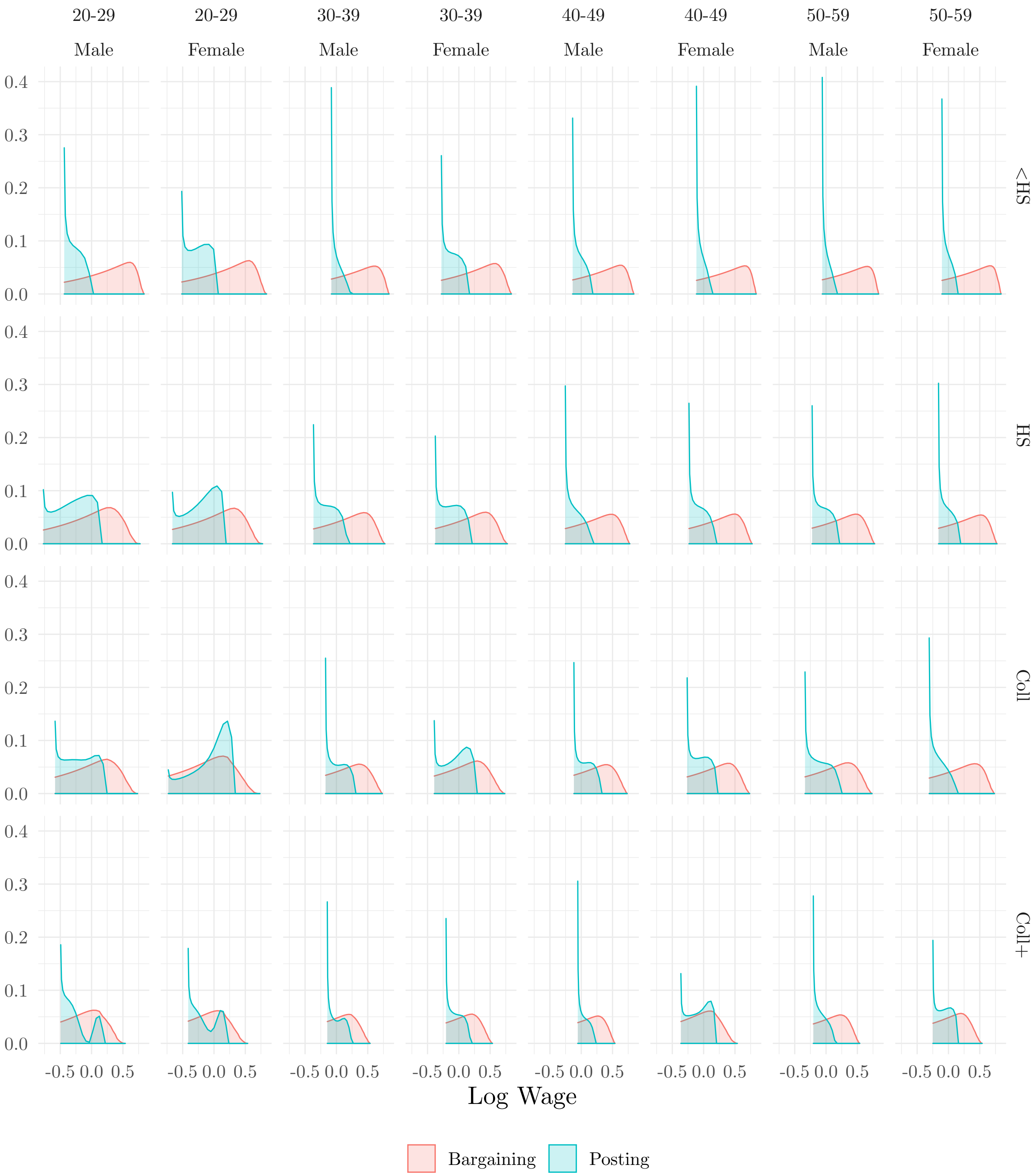
5.1 The Distribution of Wages at R and N Firms

A key mechanism of interest in the model is the interaction between R and N firms in the labor market in terms of how each firm's wage-setting strategies contribute at different locations to determining the steady state wage distribution. In order to explore this feature of the model, in Figure 7 we plot the conditional steady state log-wage densities at bargaining (R) and posting (N) firms. Several features appear that were antcipated by our theoretical analysis. According to Proposition 3, posting firms offer a wage function that begins at $\varphi(w^*) = w^*$ with a derivative that approaches zero as productivity approaches the reservation value (from above). Accordingly, the wage density at N firms often features "spikes" that are generated by these flat regions in the firm's equilibrium offer function. Furthermore, in markets where p_R is higher, the incentive to increase wages to guard against poaching by R firms intensifies near the mode of the match distribution, resulting in an inflection point in φ and a bimodal offer density ϕ, which results in a bimodal steady state wage distribution at N firms.

Figure 7 also confirms that the right tails of the wage distribution are generated by R firms, which compete with

[17] Recall these are pairs of σ and b for each of the four education categories.

Figure 7: Density of Log Wages at Firm Types



This figure shows the steady state density of log wages, calculated at estimated parameters, at bargaining and posting firms for each combination of age, sex, and education groups.

each other for workers through renegotiation. This results in two important implications for wage inequality. First, the existence of both types of firms can contribute to dispersion in wages, with N firms largely occupying the bottom of the wage distribution and R firms occupying the top. Hence, we will see in the next section that removing *either* type of firm from the labor market leads to reductions in within-market wage variance. Second, competition between R firms creates a "bargaining premium," which is a positive mean difference in wages between workers at R firms and those at N firms[18]. In Section 6 we use counterfactuals that alternatively eliminate each wage-posting strategy from the market to assess the contribution of this mechanism to several different metrics of wage inequality.

5.2 Wage Inequality and Inefficient Mobility

Relative to prior empirical work on search frictions in labor markets, allowing for heterogeneity in wage-setting protocols results in three novel empirical predictions: (1) higher rates of bargaining and renegotiation can lead to increases in *within-group* wage inequality; (2) differences across markets in rates of bargaining and renegotiation can contribute to *between-group* wage inequality due to the bargaining premium; and (3) the presence of mixed wage-setting equilibria within a market can result in *inefficient mobility.* In this section we define our statistics of interest and evaluate them at the point estimates of the parameters characterizing the models. We will quantitatively assess the contributions of differences in wage setting protocols to the wage distributions and outcomes in the counterfactual analyses discussed below.

We will focus on four statistics that measure wage inequality across various dimensions of the steady state wage distributions. At this point it is helpful to reiterate that calculations of wage statistics from the model feature *only* the contribution from search frictions, $\log(\varepsilon)$, while wages in the data are the sum of this component and permanent ability, $\log(a) + \log(\varepsilon)$. In this sense, $\log(a)$ can be thought of as the residual that explains any differences in statistics between the data and the model counterpart. We begin with the decomposition of the overall variance in log wages that is due to search frictions (ε), which is given by:

$$\mathbb{V}[\log(W_{SS})] = \underbrace{\mathbb{E}[\mathbb{V}[\log(W_{SS})|x]]}_{\text{Within-group component}} + \underbrace{\mathbb{V}[\mathbb{E}[\log(W_{SS})|x]]}_{\text{Between-group component}}$$

where x indicates the demographic group of the individual.

The starkest differences in bargaining patterns appear between gender and education categories. It is of interest to determine how much of the gender and schooling gaps in ln w are attributable to differences in search frictions between groups. To examine this question, we calculate population-weighted gender and education gaps in wages. The gender gap is calculated as:

$$\text{Gender Gap} = \frac{\sum_{x\in\text{Male}}\omega(x)\mathbb{E}[\log(W_{SS})|x]}{\sum_{x\in\text{Male}}\omega(x)} - \frac{\sum_{x\in\text{Female}}\omega(x)\mathbb{E}[\log(W_{SS})|x]}{\sum_{x\in\text{Female}}\omega(x)}$$

where $\omega(x)$ is the population weight for demographic group x calculated from the CPS sample. The education group is a similar calculation in which we divide the demographic groups into those with at least a Bachelor's degree (the top two education categories) and those with less (the bottom two categories):

$$\text{Education Gap} = \frac{\sum_{x\in\geq\text{Bachelor's}}\omega(x)\mathbb{E}[\log(W_{ss})|x]}{\sum_{x\in\geq\text{Bachelor's}}\omega(x)} - \frac{\sum_{x\in<\text{Bachelor's}}\omega(x)\mathbb{E}[\log(W_{SS})|x]}{\sum_{x\in<\text{Bachelor's}}\omega(x)}$$

[18]These wage premia are also detectable in the data of Hall and Krueger (2012), although not directly comparable due to selection on unobserved ability, which we do not observe.

Table 2: Baseline Statistics from the Estimated Model

	Baseline	% of Population Value
$\mathbb{E}[\mathbb{V}[\log(W)\|X]]$	0.03	10.43
	(0.001)	(0.51)
$\mathbb{V}[\mathbb{E}[\log(W)\|X]]$	0.006	6.29
	(0.002)	(1.54)
Gender Wage Gap	0.002	0.82
	(0.009)	(4.5)
Education Wage Gap	0.082	15.7
	(0.022)	(4.32)
Inefficient Mobility (%)	14.87	-
	(0.26)	-

This table reports the statistics of interest outlined in Section 5.2 for the estimated model. $\mathbb{E}[\mathbb{V}[\log(W)|X]]$ is referred to as within-group inequality, while $\mathbb{V}[\mathbb{E}[\log(W)|X]]$ is referred to as between-group inequality. The second column reports statistics as a percentage of their population value in the CPS data. Standard errors are calculated using 100 bootstrap samples.

Finally, we will also examine the fraction of all job-to-job moves in which a worker leaves an N firm to join a less productive R firm. This rate can be written as:

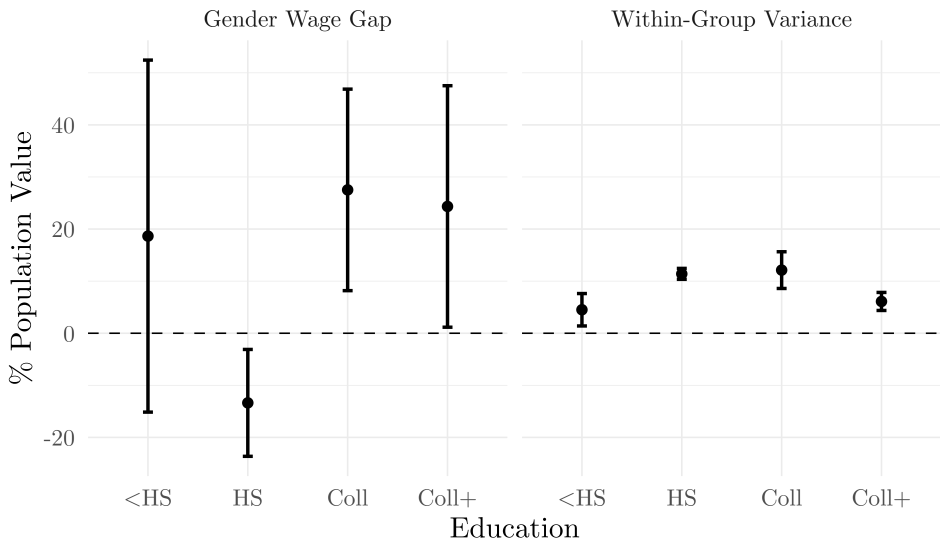
$$\frac{\lambda_E p_R \int_{w^*} g_N(w)(F_\theta(\varphi^{-1}(\theta)) - F_\theta(w))dw}{EE}. \tag{13}$$

where EE is the steady state rate of employer-employer transitions, given in equation (12).

Table 2 reports the values of these statistics. In order to assess the magnitude of the results, in the second column we normalize by the population value of each statistic calculated in the CPS data (with the exception of inefficient mobility which is unknown).[19] Table 2 provides a number of introductory quantitative insights. As we outlined in Section 3.4, our focus is on residual wage dispersion, which is the portion of wages explained by frictions in the labor market rather than by individual differences in productivity. Thus, when we compare our baseline calculations to their population values, one should interpret these as the percentage of the overall variation in $\ln w$ that can be attributed to search frictions. For example, we find that search frictions can explain nearly 10 percent of within-group variation in log wages, and 6.3 percent of the between-group log wage variance. In terms of wage gaps, we find that frictional wage dispersion can account for 15 percent of the education log wage gap, but essentially none of the gender gap in log wages. Although we found this latter result to be surprising, on closer inspection we find that it masks important amounts of heterogeneity across education groups. In Figure 8 we compute the same weighted averages for gender gaps and within-group log wage variance separately by education group. Here we find that for college graduates and above, frictional wage dispersion across markets can account for between 20 and 30 percent of the population gender gap. Interestingly, our estimates also suggest that frictional wage dispersion is offsetting gender gaps in wages for high school graduates. Turning to mobility

[19] These population values include individual ability, while the computed values of our statistics only include the contribution from search frictions, represented by ε. Our measure of inefficient mobility is independent of a given the specification of our model.

Figure 8: Wage Inequality by Education Group

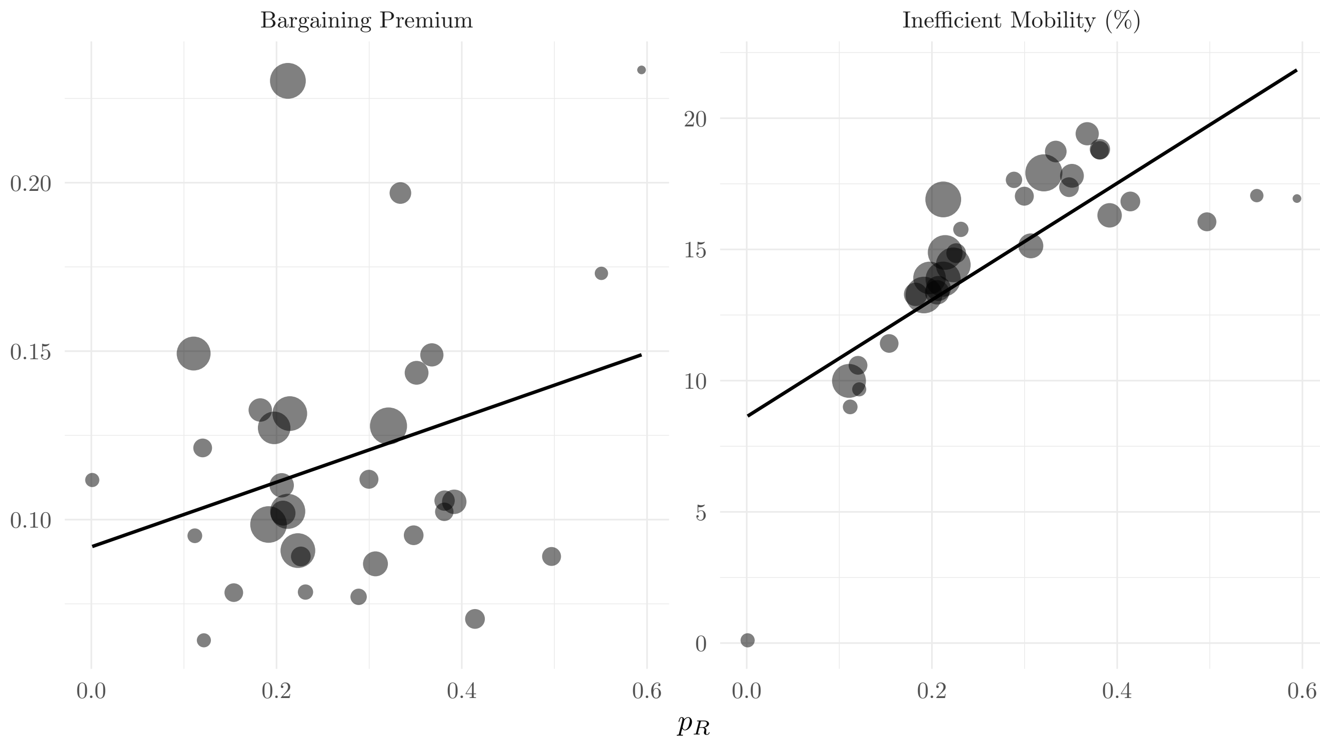


This figure shows within-group inequality ($\mathbb{E}[\mathbb{V}[\log(W)|X]]$) and gender wage gaps calculated separately for each education level in the estimated model. All statistics are reported as a percentage of their population value in the CPS data. 95% confidence intervals are shown using 100 bootstrap samples.

we find that, in the steady state, 15 percent of job moves can be considered inefficient in the sense that the worker is moving from a more productive match at an N firm to a less productive much at an R firm. One would reasonably expect that this rate is higher for markets in which the two firms are more likely to interact, which is confirmed by Figure 9.

While this analysis can speak to the overall contribution of search frictions to differences in wage inequality, it does not offer a way to disentangle the effect of differences in wage-setting from other market primitives that shape the wage distribution, such as the rate parameters $(\lambda_E, \lambda_U, \delta)$ and worker bargaining power, α. It also does not tell us about the relevance of inefficient mobility to overall output. In the next section, we will use two counterfactuals to evaluate the contribution of differences in wage-setting choices across markets to these calculations. We will also calculate the potential output gains from removing inefficient mobility from each market. Before moving to this analysis, in Figure 9 we plot the relationship between equilibrium values of p_R and bargaining premia, as well as the relationship between p_R and rates of inefficient mobility, across labor markets. Unsurprisingly, we see a remarkably tight relationship between p_R and the rate of inefficient moves, since increases in p_R increase the probabilty of interactions between N and R firms, at least for the values of p_R observed in the data. In theory of course, this relationship is U-shaped, which explains why a flattening out of the rate appears to show for higher values of p_R. We see that the rate of inefficient mobility varies between 10 and 20 percent in the estimated model, suggesting that the potential output gains from forcing all firms to adopt the same wage-setting practices can be expected to vary substantially across markets. Figure 9 also shows substantial variation in the bargaining wage premium, ranging between 5 and 25 percent across markets. This relationship is also positive, although there are clearly other market level parameters (such as productivity dispersion σ and κ) that determine the size of the premium.

Figure 9: p_R vs Inefficient Mobility



This figure plots the equilibrium fraction of R firms (p_R) in each market against the bargaining premium (left panel, defined as the mean difference in log wages paid at R firms relative to N firms) and the rate of inefficient job switches (right panel, defined in equation (13)). Point sizes are proportional to the number of observations for each demographic group.

Table 3: The Impacts of Wage-Setting Mandates on Inequality and Efficiency

	% of Data Value		% of Model Baseline	
	$p_R = 0$	$p_R = 1$	$p_R = 0$	$p_R = 1$
$\mathbb{E}[\mathbb{V}[\log(W)\vert X]]$	-1.21	-1.3	-11.64	-12.47
	(0.06)	(0.16)	(0.49)	(1.23)
$\mathbb{V}[\mathbb{E}[\log(W)\vert X]]$	-0.58	-4.56	-9.19	-72.48
	(0.36)	(0.83)	(4.2)	(6.38)
Gender Wage Gap	-6.78	-2.39	-823.12	-290.14
	(1.27)	(1.96)	(3122.2)	(870.69)
Education Wage Gap	-3.09	-11.13	-19.72	-70.89
	(0.65)	(2.02)	(6.12)	(17.26)
Inefficient Mobility (%)			-100.0	-100.0
			(0.0)	(0.0)

This table reports the effect of wage posting ($p_R = 0$) and bargaining ($p_R = 1$) mandates on each statistic of interest. The first column reports effect sizes as a percentage of the statistic's value in the CPS data. The second column reports effect sizes as a percentage of the estimated model's baseline value. Standard errors are calculated using 100 bootstrap samples.

6 Counterfactual Analysis

6.1 The Contribution of Differences in Wage-Setting to Wage Inequality and Output

We next use the estimated model to perform a pair of counterfactuals designed to reveal how differences in wage-setting protocols may contribute to overall inequality in wages. We consider the effect of two policies, which alternatively mandate that only wage-posting or bargaining be used to determine wages. This is achieved by imposing that $p_R = 0$ (we call this the posting mandate) or $p_R = 1$ (the bargaining mandate). Our motivations for these exercises are twofold. From a positive perspective, they are necessary to evaluate the specific role played by wage-setting protocols in shaping wages and output. From a normative perspective, these counterfactuals are a first step to evaluate policies that seek to regulate the wage-setting process. As we discussed in the introduction, such policies are gaining increasing political traction.

In order to evaluate each counterfactual we set p_R to its mandated level (0 or 1), recompute reservation wages and, when $p_R = 0$, the equilibrium wage offer function φ. All statistics are evaluated at the new steady state equilibrium.[20] To isolate mechanisms, we first do this by keeping contact rates fixed, and then later check for robustness of the results when contact rates are determined in equilibrium through vacancy posting.

Table 3 reports the impact of both counterfactuals on each of the population weighted statistics reported in Table 2, where each statistic is reported as a percentage of its population estimate in the data. The overall policy effects on wage inequality are statistically significant, but somehwat underwhelming in terms of magnitudes relative to their total population values in the data. Both mandates reduce within-group wage inequality by 11 to 12 percent relative to the

[20] We do not solve for transitional dynamics due to the intractability of the problem.

Table 4: The Impacts of Wage-Setting Mandates on Inequality and Efficiency: Endogenous Contact Rates

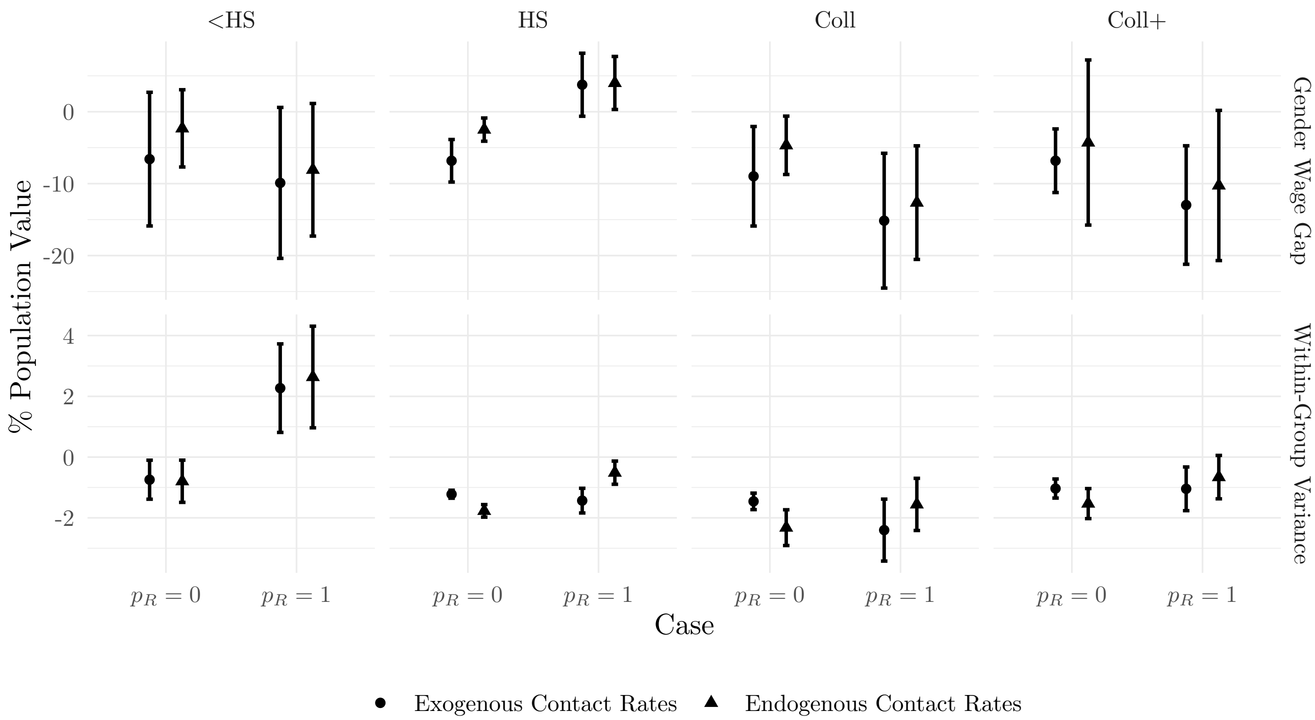
	% of Data Value		% of Model Baseline	
	$p_R = 0$	$p_R = 1$	$p_R = 0$	$p_R = 1$
$\mathbb{E}[\mathbb{V}[\log(W)\|X]]$	-1.79	-0.5	-17.11	-4.78
	(0.1)	(0.14)	(0.55)	(1.3)
$\mathbb{V}[\mathbb{E}[\log(W)\|X]]$	-0.6	-4.24	-9.57	-67.38
	(0.43)	(0.78)	(3.78)	(5.93)
Gender Wage Gap	-2.95	-1.29	-358.46	-156.08
	(0.95)	(1.67)	(1468.63)	(304.85)
Education Wage Gap	-1.72	-9.77	-10.93	-62.27
	(0.46)	(1.63)	(3.25)	(14.52)
Inefficient Mobility (%)			-100.0	-100.0
			(0.0)	(0.)
Contact Rates (λ_U)			10.29	-16.1
			(2.35)	(1.6)

This table reports the effect of wage posting ($p_R = 0$) and bargaining ($p_R = 1$) mandates on each statistic of interest. The first column reports effect sizes are a percentage of the statistic's value in the CPS data. The second column reports effect sizes as a percentage of the estimated model's baseline value. Standard errors are calculated using 100 bootstrap samples.

model baseline. However, given that the model explains only 10 percent of what is observable in the data (neglecting ability, a), this translates to a 1.1 to 1.2 percent reduction in total within-group wage inequality. We find the overall pattern of results is unchanged when we allow for endogenous contact rates, as show in Table 4. We find that posting mandates lead to an increase in contact rates, while bargaining mandates lead to a decrease in contact rates. This holds in general because profits for both types of firms are decreasing in p_R. Most notably, the effect on contact rates implies that the effect of these policies on residual wage dispersion is exacerbated relative to the simple counterfactual for posting mandates and muted for the case of bargaining mandates. This contrast can be understood through effects on κ. Increases in contact rates lead to increases in κ, which in turn leads to increases in frictional wage dispersion.

Turning to inequalities between groups, a posting mandate reduces the gender gap by almost 7 percent of its population value and reduces the education gap by about 3 percent. A bargaining mandate, on the other hand, reduces the education gap by 11 percent of its population value and has no statistically significant effect on the gender gap. Since we document a reasonable degree of heterogeneity in gender gaps and within-group wage variance among education categories, Figure 10 documents the effect of each counterfactual on these statistics as a percentage of their data value. The gender gap impacts are reasonably consistent at between 10 and 15 percent of the total data value, despite the model having explained a much higher fraction of this gap in the baseline for the higher two education groups. Reductions in within-group inequality from posting mandates are consistently between 1 and 2 percent of the total within-group variance in the data, while bargaining mandates have heterogeneous effects. In particular, bargaining leads to a 2 percent *increase* in within group

Figure 10: Effect of Wage Mandates on Wage Inequality and Gender Gaps



This figure shows the effect of Posting ($p_R = 0$) and Bargaining ($p_R = 1$) mandates on within-group inequality ($\mathbb{E}[\mathbb{V}[\log(W)|X]]$) and gender wage gaps for each education level. All statistics are reported as a percentage of their population value in the CPS data.

inequality for high school dropouts.

6.2 Welfare Effects of Wage-Setting Mandates

Our counterfactual analysis so far suggests that regulating wage-setting practices has the potential to reduce wage inequality, and conversely replicates the finding in empirical work (Biasi and Sarsons, 2021) that allowing for bargaining can lead to increases in gender gaps and wage inequality more generally. However, we have not yet examined the overall implications for wages and welfare, which we consider in this section. This is particularly important because policies that are designed to regulate wage-setting are usually motivated by the goal of protecting the interests of workers.

Figure 11 depicts the results, calculated separately by education group. It indicates that posting mandates lead uniformly to small reductions in welfare (on the order of 1 to 3 percent of baseline consumption), while bargaining mandates result in somewhat more sizeable increases in welfare (ranging between 5 to 15 percent of baseline consumption). These results are robust to allowing for adjustment in contact rates through firm vacancy creation, although effects are slightly muted across all cases when allowing for endogenous contact rates. Under the bargaining mandate, firms are forced to compete with each other more often for workers through renegotiation, leading to sizeable increases in worker wages in the steady state. These normative findings are significant, but limited in the sense that we do not consider the possibility of redistributing gains in output from firms to workers. Conversely, when R firms are removed from the market under posting mandates, this leads to an equilibrium decrease in the wage offer function φ since posting firms face less competition for retention from poaching R firms. This leads to the welfare losses we observe in Figure 11. The patterns observed are the same whether contact rates are assumed to be fixed or endogenous, although,as is typical, increases in the welfare of workers from a strict bargaining regime are more muted.

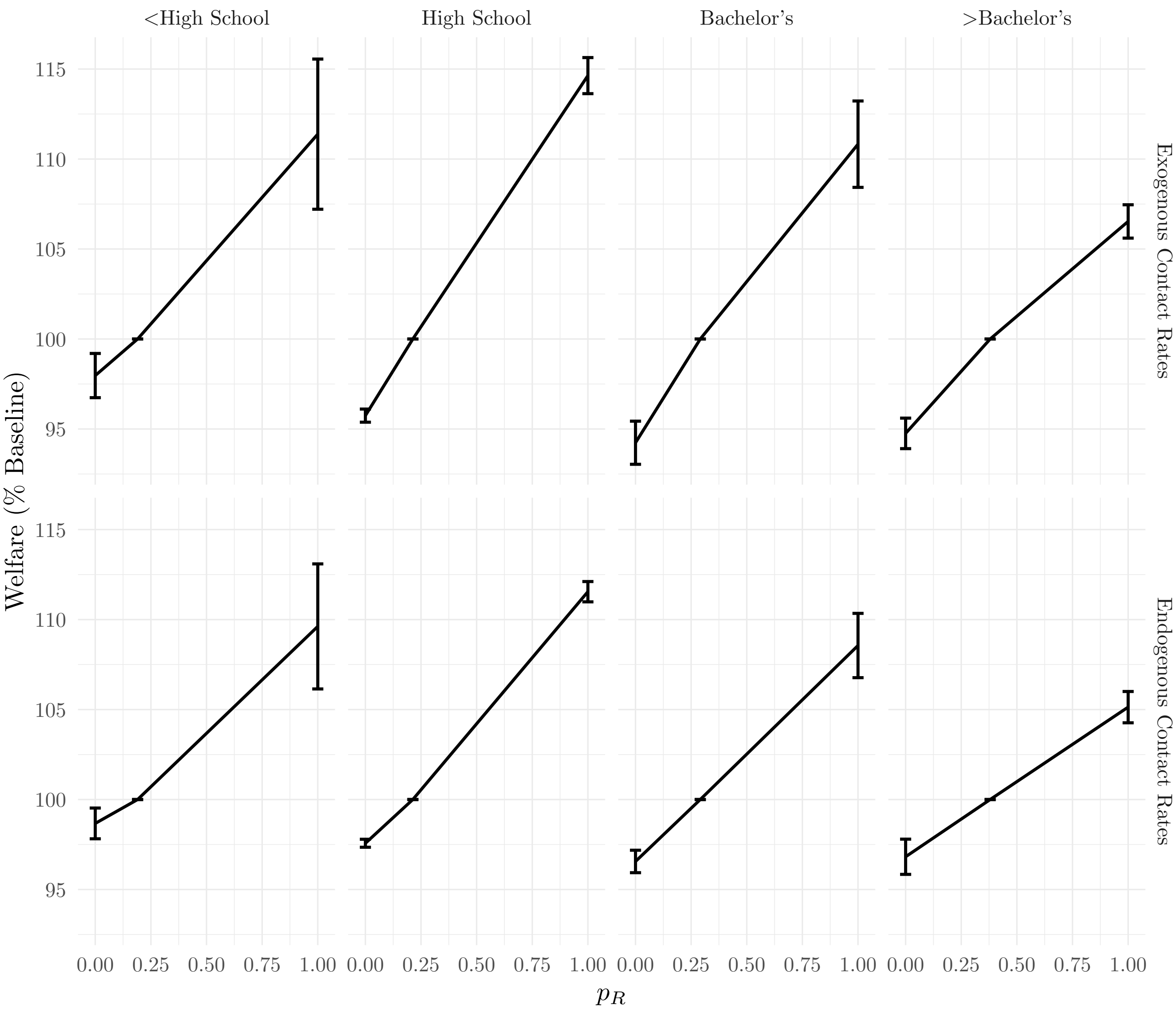
6.3 Quantifying Inefficient Mobility

Both bargaining and posting mandates eliminate inefficient mobility, since both policies impose homogenous wage-setting practices across firms within a market, ensuring that workers only move to jobs with higher match productivities. While we have calculated that 15 percent of all job moves are inefficient in the baseline model, it is not clear what the consequences of this are for output. We complete the analysis by evaluating the effect of wage-setting mandates on output for the case of both fixed and endogenous contact rates.

Figure 12 reports the results, calculated separately as a population weighted average for each education group. For all groups except high school dropouts, posting mandates result in output gains of percent relative to the baseline. Efficiency gains from bargaining mandates tend to be higher ranging between 2 and 3 percent of baseline output across education groups. However, these results are not robust to allowing for endogenous contact rates. In particular, forcing firms to compete via bargaining and renegotiation leads to a reduction in vacancy posting that outweighs the gain in output from eliminating inefficient mobility. By contrast, posting mandates result in slightly greater output effects due to the positive impact of the mandate on vacancy creation. These results suggest that while bargaining mandates lead to positive welfare effects for workers, posting mandates may be preferable to a planner with access to redistributive fiscal instruments.

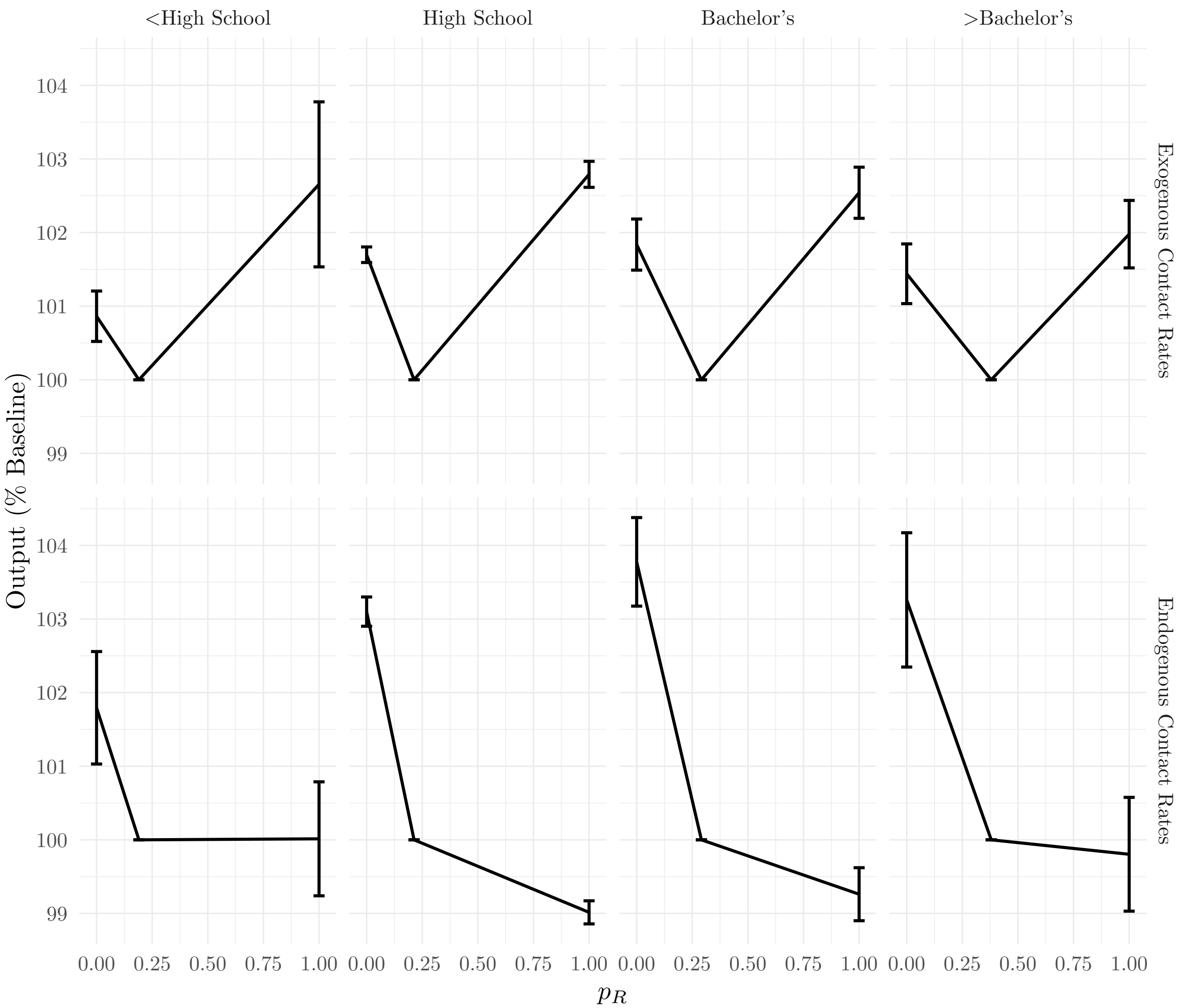
Mobility decisions are not the *only* source of inefficiency in the model. Even when wage-setting is homogenous in a market, job acceptance decisions out of unemployment may be inefficient. The worker does not internalize the full social value of continuing to search for jobs, and typically has a reservation wage that is lower than the planner's. In this sense, changes in output also reflect changes in the worker's reservation wage. However Figure 13, which plots the initial

Figure 11: Welfare Effects of Wage Mandates



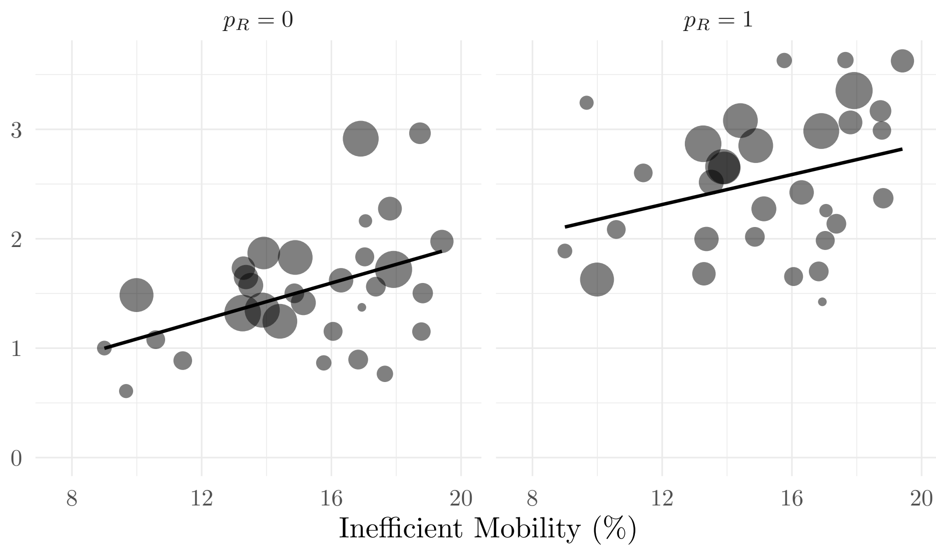
This figure shows the weighted average of worker welfare by education level, computed as a percentage of welfare at the estimated baseline. Error bars show 95% confidence intervals for each estimate relative to baseline. On the x-axis, points at $p_R = 0$ correspond to the mandated posting counterfactual while points at $p_R = 1$ correspond to the mandated bargaining counterfactual. Points with $p_R \in (0, 1)$ represent the weighted average of the estimated p_R for each demographic within an education category.

Figure 12: Output Losses from Inefficient Mobility



This figure shows the weighted average of total output by education level, computed as a percentage of output at the estimated baseline. Error bars show 95% confidence intervals for each estimate relative to baseline. On the x-axis, points at $p_R = 0$ correspond to the mandated posting counterfactual while points at $p_R = 1$ correspond to the mandated bargaining counterfactual. Points with $p_R \in (0, 1)$ represent the weighted average of the estimated p_R for each demographic within an education category.

Figure 13: Inefficient Mobility vs Output Gains



This figure plots, for each market, the initial rate of inefficient job moves (given in (13)) against the output gain from bargaining and posting mandates. Point sizes are proportional to sample size for each market. The positive relationship between these variables indicates that removing inefficient mobility is relevant for observed output gains in each market.

rate of inefficient mobility against the output gains in each market in partial equilibrium, confirms that eliminating this particular market feature plays an important role in explaining the overall effect on output. Markets with higher rates of inefficient mobility experience the larger gains in output from wage-setting mandates.

It is worth noting that, as in all models of undirected search with vacancy posting, a congestion externality implies that firms generally do not make efficient vacancy creation decisions. We leave investigation of the significance of this channel to future work.

7 Conclusion

We studied labor markets in which firms' choices of how to set wages is endogenous to market primitives and government policies. While the space of potential wage-setting protocols is vast, we focused on two benchmark protocols in the literature by allowing firms to choose between bargaining with renegotiation and posting non-negotiable wage offers. In a simple benchmark case with no unobservable heterogeneity in worker productivity with segmented labor markets we derived a testable prediction for the cross-sectional prevalence of bargaining and renegotiation in the data and validated the model's mechanism by showing that this pattern does indeed appear. We extended this benchmark case to allow for heterogeneous worker and job productivities, deriving simple and interpretable rules for wage determination and mobility decisions. We ended the quantitative analysis by evaluating the welfare and efficiency consequences of policies that regulate wage setting. We find that differences across workers in the prevalence of bargaining can explain statistically meaningful components of gender and education wage gaps, as well as residual wage dispersion. While wage posting mandates reduce wage inequality, they also lead to losses in worker welfare and gains in aggregate output in equilibrium.

These exercises highlight the need for an equilibrium framework for wage-setting in order to evaluate the impacts of labor market policies. This applies not only to policies that seek to regulate the wage-setting process (in the spirit of recent salary history bans), but also more generally in any context in which it is not reasonable to assume that wage-setting practices are policy invariant (Marschak, 1974; Lucas, 1976).

A Data Sources and Construction

There are three sources of data used in the paper. We use the Current Population Survey for information on unemployment rates, wages, and labor market transitions. We use the survey data from Hall and Krueger (2012) for information on rates of bargaining and wage posting, and we use the Survey of Consumer Expectations for information on rates of renegotiation.

We calculate statistics separately by demographic groups that we define consistently across all three datasets as combination of age groups, sex, and education groups. Age groups are defined by the ranges 20-29, 30-39, 40-49, 50-59. Our education groups are those with less than High School or some equivalent, those with at least High School or equivalent but less than a Bachelor's degree, those with a Bachelor's degree, and those with more than a Bachelor's degree.

A.1 Hall and Krueger (2012)

We use two questions from this survey dataset[21]. The main question (`Q34d`) we use, which we take as evidence of bargaining, is worded as follows:

> "When you were offered your current job, did your employer make a "take-it-or-leave-it" offer or was there some bargaining that took place over the pay?"

It is important to clarify our interpretation of this question since it is common for bargaining games, such as in Rubinstein (1982), to be resolved in the first period of play. However, even if workers accept the employer's first offer in equilibrium, it is important to note that offers in such games are not "take-it-or-leave-it" and that both players in the game are aware that counteroffers may transpire. In this sense, it is not inconsistent for players in the game to report to having participated in a bargaining game even if the first wage offer is accepted. Furthermore, they would not report the first offer as "take-it-or-leave-it" after having participated in such a game.

We calculate the mean rate of bargaining using this variable for each demographic group, which is used in Figure 1, Figure 3, and as moments in the estimation routine described in Section 4.2.

In Figure 14 below we augment Figure 1 to also report rates of wage posting, which we construct using survey item `Q34b`:

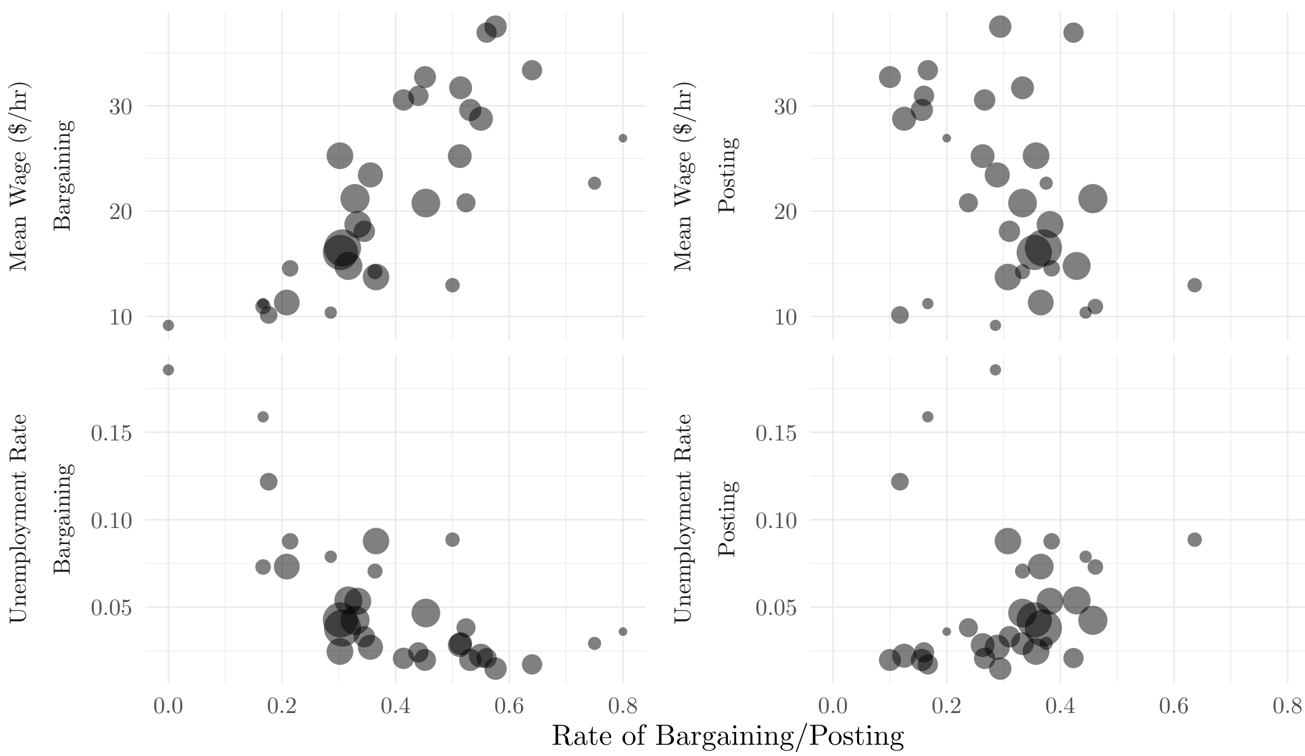
> "At the time that you were first interviewed for your job, did you already know exactly how much it would pay, have a pretty good idea of much it would pay, or have very little idea of how much it would pay if you got it?"

We designate the wage as having been posted if the individual reports knowing exactly how much the job would pay. There is some ambiguity here since there is no explicit application decision in the model and workers do not necessarily

[21] Available at `https://www.openicpsr.org/openicpsr/project/114257/version/V1/view`

know the wage they will earn, since this is drawn from a distribution in equilibrium. A cleaner interpretation of this survey question would require additional modelling assumptions.

Figure 14: The Relationship Between Bargaining/Posting and Wages/Unemployment



This figure shows the relationship between wages and unemployment and rates of bargaining found in the survey of Hall and Krueger (2012). Each point represents a combination of age category, sex, and education category. Age categories are 20-29, 30-39, 40-49, and 50-59. Education categories are high school dropouts, high school graduates or equivalent, less than four years of college, and four or more years of college. Rates by demographic cell are matched to labor market statistics calculated using the *Current Population Survey* (CPS). Appendix A provides additional details. Points are proportional to sample size.

A.2 Survey of Consumer Expectations

The data on renegotiation used in the construction of Figure 3 is taken from the Federal Reserve Bank of New York's *Survey of Economic Expectations*.[22]. Our sample consists of all employed individuals between the ages of 20 and 60. Variables with information on age, sex, and education are taken from the core survey, and are linked for each individual in the labor market survey. In this latter survey, individuals report their subjective evaluation of the probability that their current employer would match a counteroffer (survey item `OO2f`). This results in 1,455 non-missing observations from July and November 2015. By demographic cell we calculate the average fraction of individuals who report a positive probability that their employer would match an outside offer. We link each of these means with the corresponding demographic cell statistics calculated in the 2015 sample of the CPS, described below.

[22]http://www.newyorkfed.org/microeconomics/sce

A.3 Current Population Survey

All of our CPS data was retrieved using the Minnesota Population Center's IPUMS integrated database (Flood, King, Rodgers, Ruggles, and Warren, 2020). We construct two samples: one from January through December 2008 and one from January through December 2015. In both cases, we select individuals between the ages of 20 and 60 who are in the labor force (variable LABFORCE). We define individuals to be unemployed (using the variable EMPSTAT) if they report being at work as recently as last week. We code employment transitions by linking individuals across months using IPUMS' individual identifier (CPSIDP). In particular, we denote individuals as having an EE transition if they report being employed in consecutive months, but no longer having the same employer as last month (variable EMPSAME).

To construct wage moments, we select the subset of individuals from the Outgoing Rotation Group subsample of the monthly survey. We use reported hourly wage (HOURWAGE) for those who report being paid by the hour, and impute wages as usual weekly earnings (EARNWEEK) divided by usual weekly hours at their main job (UHRSWORK1). Our "steady state" sample of wages is defined as all wage observations in the cross-section, while our sample of wages accepted out of unemployment is all observations in which the individual reported being unemployed in the previous month.

We link means from the latter sample (2015) by demographic cell to mean rates of bargaining from the SCE. Conversely, we link means from the former sample (2008) by demographic cell to their counterparts in the Hall and Krueger (2012) data.

B Additional Details for the Homogenous Productivity Model

B.1 Steady State Distributons

Given an offer distribution Φ, a worker can be in several states. A fraction U will be unemployed, while the remaining fraction $1-U$ will be employed. Employed workers can have three distinct states, a fraction M_N will be at N firms. Another fraction M_R will be at their first R firm, while the remaining fraction M_{2R} will have met another R firm on the job (bidding up their wage to z). By balancing flow equations, we get:

$$U = \frac{\delta}{\delta + \lambda_U} \tag{14}$$

$$M_N = \frac{\delta(1-p_R)}{\delta + \lambda_E p_R} = \frac{1-p_R}{1+\kappa p_R} \tag{15}$$

$$M_R = \frac{(1+\kappa)}{(1+\kappa p_R)^2} \tag{16}$$

$$M_{2R} = \frac{\kappa p_R^2(1+\kappa)}{(1+\kappa p_R)^2} \tag{17}$$

$$\tag{18}$$

Workers at N firms have a steady state distribution over their wages, G_N, while workers at their first R firm since unemployment have a steady state distribution over the outside option used to bargain their wage, G_R. Balancing flow equations gives:

$$G_N(w) = \frac{(\delta + \lambda_E p_R)\Phi(w)}{\delta + \lambda_E p_R + \lambda_E(1-p_R)\tilde{\Phi}(w)} = \frac{(1+\kappa p_R)\Phi(w)}{1+\kappa p_R + \kappa(1-p_R)\tilde{\Phi}(w)} \tag{19}$$

$$G_R(w) = \frac{(1+\kappa p_R)^2}{(1+\kappa p_R + \kappa(1-p_R)\tilde{\Phi}(w))^2} \tag{20}$$

B.2 Equilibrium Wage-Setting

Let $e(w)$ be the exit rate from an N-firm offering w:

$$e(w) = \delta + \lambda_E p_R + \lambda_E(1 - p_R)\tilde{\Phi}(w)$$

Similarly let $e_\alpha(w)$ denote

$$e_\alpha(w) = \delta + \lambda_E p_R \alpha + \lambda_E(1 - p_R)\tilde{\Phi}(w)$$

For N-firms, the value of offering w is:

$$\Pi_N(w) = \left[\frac{\delta}{\delta + \lambda_E} + \frac{\lambda_E}{\delta + \lambda_E}\frac{\delta(1 - p_R)\Phi(w)}{e(w)}\right]\frac{z - w}{e(w)}$$

which simplies to:

$$\Pi_N(w) = \frac{z - w}{1 + \kappa}\frac{\delta + \lambda_E}{e(w)^2}$$

As in Burdett and Mortensen (1998), in equilibrium firms must be indifferent across wage choices in the support of Φ. Thus, restricting $\Pi_N(w) = \Pi_N(w^*)$ gives the solution:

$$\Phi(w) = \frac{1 + \kappa}{\kappa(1 - p_R)}\left(1 - \sqrt{\frac{z - w}{z - w^*}}\right)$$

and for the hazard function:

$$\tilde{\Phi}(w) = \frac{1}{\kappa(1 - p_R)}\left[(1 + \kappa)\sqrt{\frac{z - w}{z - w^*}} - 1 - \kappa p_R\right]$$

This expression gives us the upper bound on the support of wages:

$$\overline{w} = \left[1 - \left(\frac{1 + \kappa p_R}{1 + \kappa}\right)^2\right]z + \left(\frac{1 + \kappa p_R}{1 + \kappa}\right)^2 w^* \tag{21}$$

and note the limits for p_R. Now let's solve for Π_R. With probability $\frac{\lambda_U U}{\lambda_U U + \lambda_E(1-U)}$ the worker is unemployed, in which case the firm gets a fraction $(1 - \alpha)$ of the surplus $T - V_N(w^*)$ where T is the total value of the match and $V_N(w^*)$ is the value of unemployment. With probability equal to $\frac{\lambda_E(1-U)}{\lambda_U U + \lambda_E(1-U)} \times M_N$ the worker is at an N firm, distributed over wages according to G_N. In this case the firm's profit is $(1 - \alpha)(T - V_N(w))$. If the firm meets a worker at another R firm, the wage is bid up to z and the firm receives zero profit. Putting this together we get the expression:

$$\Pi_R = \frac{(1 - \alpha)}{\delta + \lambda_E}\left[\delta(T - V_N(w^*)) + \lambda_E\frac{\delta(1 - p_R)}{\delta + \lambda_E p_R}\int(T - V_N(w))dG_N(w)\right]$$

Note that:

$$\delta V_N(w) = w + \lambda_E p_R \alpha(T - V_N(w)) + \lambda_E(1 - p_R)\int(V_N(w') - V_N(w))d\Phi(w)$$

which gives:

$$V_N'(w) = \frac{1}{\delta + \lambda_E \alpha p_R + \lambda_E(1 - p_R)\tilde{F}(w)} = \frac{1}{e_\alpha(w)}$$

Now note that using integration by parts:

$$\int V_N(w)dG_N(w) = V_N(\overline{w}) - \int\frac{G_N(w)}{e_\alpha(w)}dw \tag{22}$$

$$= \int\frac{\tilde{G}_N(w)}{e_\alpha(w)}dw + V_N(w^*) \tag{23}$$

Simarly the fundamental theorem gives:

$$V_N(\overline{w}) = \int \frac{1}{e_\alpha(w)} dw + V(w^*) \tag{24}$$

Substituting (22) and (24) into the expression for Π_R, and using (21) to relate $z - \overline{w}$ to $z - w^*$, gives:

$$\Pi_R = (1-\alpha)\delta \left[\frac{z - w^*}{(\delta + \lambda_E p_R)(\delta + \lambda_E p_R \alpha)} \frac{(\delta + \lambda_E p_R)^2}{(\delta + \lambda_E)^2} + \int_w^{\tilde{w}} \frac{1}{e(w)e_\alpha(w)} dw \right].$$

The integrand simplifies to:

$$\int_{w^*}^{\tilde{w}} \frac{1}{(\delta + \lambda_E)\sqrt{\frac{z-w}{z-w^*}}(\lambda p_R(\alpha - 1) + (\delta + \lambda_E)\sqrt{\frac{z-w}{z-w^*}})} dw$$

Now using a change of variables with $u = \sqrt{\frac{z-w}{z-w^*}}$, we get:

$$\Pi_R = \frac{(1-\alpha)\delta(z - w^*)}{(\delta + \lambda_E^2)} \left[\frac{1 + \kappa p_R}{1 + \kappa p_R \alpha} + 2 \log \left(\frac{1 + \kappa - \kappa p_R(1-\alpha)}{1 + \kappa p_R \alpha} \right) \right].$$

Comparing this to

$$\Pi_N(w^*) = \frac{\delta}{(\delta + \lambda_E)^2}(z - w^*)$$

shows us that the equilibrium fraction p_R depends only on κ and α. The condition is:

$$(1-\alpha) \left[\frac{1 + \kappa p_R}{1 + \kappa p_R \alpha} + 2 \log \left(\frac{1 + \kappa - \kappa p_R(1-\alpha)}{1 + \kappa p_R \alpha} \right) \right] = 1$$

We can use this to sketch out contours at different values of p_R.

B.3 Flows

Note that if EE, EU, and UE are the empirical flows, there is a one-to-one mapping between λ_U and UE, and between δ and EU. Finally, for EE, it will be useful first to derive the steady state distribution of $u = \Phi(w)$. We get:

$$(1 - M_N)G_N(u) = \frac{(1 - p_R)u}{1 + \kappa p_R + \kappa(1 - p_R)(1 - u)}$$

And it follows that the steady state density is:

$$(1 - M_N)g_N(u) = \frac{(1 - p_R)(1 + \kappa)}{(1 + \kappa p_R + \kappa(1 - p_R)(1 - u))^2}$$

We assume that all interactions between R firms, and between R firms and N firms result in a job-to-job transition. Thus, the steady state rate is:

$$EE = \lambda_E \left(p_R + (1 - p_R)^2(1 + \kappa) \int_0^1 \frac{1 - u}{(1 + \kappa p_R + \kappa(1 - p_R)(1 - u))^2} du \right)$$

which solves to[23]

$$EE = \lambda_E \left(p_R + \frac{1 + \kappa}{\kappa^2} \log \left(\frac{1 + \kappa}{1 + \kappa p_R} \right) - \frac{1 - p_R}{\kappa} \right)$$

[23] In order to solve we use that the antiderivative of $(1 - x)/(a + b(1 - x))^2$ is $b^{-2}[a/(a + b(1 - x)) + \log(a + b(1 - x))]$

B.4 Reservation Wage

First we note that

$$T - V_N(w^*) = \frac{z-b}{\rho+\delta}$$

and so we can derive the reservation wage equation:

$$w^* = b + (\lambda_U - \lambda_E)\left[p_R\alpha\frac{z-b}{\delta} + (1-p_R)\int_{w^*}(V(w) - V(w^*))d\Phi\right]$$

The usual integration by parts trick gives:

$$\int_{w^*}(V(w) - V(w^*))d\Phi = \int_{w^*}\frac{\tilde{\Phi}(w)}{\delta + \lambda_E\alpha p_R + \lambda_E(1-p_R)\tilde{\Phi}(w)}dw$$

while a change of variables $u = \tilde{\Phi}(w)$ gives us that this integral is equal to

$$(z-w^*)\frac{2\kappa(1-p_R)}{\delta(1+\kappa)^2}\int_0^1 \frac{u\kappa(1-p_R) + 1 + \kappa p_R}{1 + \kappa p_R\alpha + \kappa(1-p_R)u}du.$$

Using the antiderivative for the integrand above, we update the reservation wage equation:

$$w^* = b + \frac{\lambda_0 - \lambda_E}{\delta}\left[p_R\alpha(z-b) + (z-w^*)2\kappa\left(\frac{1-p_R}{1+\kappa}\right)^2\left(1 - \frac{p_R(1-\alpha)}{1-p_R}\log\left(\frac{1+\kappa p_R\alpha}{1+\kappa(1-p_R+p_R\alpha)}\right)\right)\right]$$

Hence, we can write the reservation wage as a linear combination of b and z.

B.5 Equilibria with Alternative Wage Contracts

In this section we consider the determination of equilibrium when the firm is able to use an additional wage-setting strategy. The two options we consider are (1) a posted fee contract in the spirit of Stevens (2004) and (2) a posted wage with the opportunity to later renegotiate. We will show that either wage-setting protocol being available results in homogenous wage-setting with $p_N = p_R = 0$. In both cases, the logic is identical: these options allow for non-negotiable offers while still maximizing the joint value of the match. We can therefore show that either contract is strictly preferable to hiring a worker using a non-negotiable wage (which does not maximize the joint value of the match), implying that $p_N = 0$. With $p_N = 0$, only unemployed workers can be successfully hired. Since both alternatives considered allow firms to post a wage that claims the full surplus of the match from unemployed workers, this is preferable to being an R firm that awards a fraction α of that surplus to the worker. Defining $T_N(w) = \Pi_N(w) + V_N(w)$ as the joint value of an N firm match, the lemma below is useful for both cases.

Lemma 1. *If $T_N(w)$ is the joint value of a match to an N firm, and T is the joint social value of the match, then $T_N(w) < T$.*

Proof. Let $F_V(\cdot|w)$ be the endogenous distribution of offered values for the worker, conditional on their current wage, w. Since T is the maximum possible joint value of a match, this is an upper bound of the support of F_V. We have

$$(\rho+\delta)V_N(w) = w + \lambda_E\int_{V_N(w)}^T (v - V_N(w))dF_V(v|w) + \delta V_U$$

and so

$$(\rho+\delta)T_N(w) = z + \lambda_E\int_{V_N(w)}^T (v - T_N(w))dF_V(v|w) + \delta V_U.$$

Define the contraction mapping:

$$\mathcal{T}[f](w) = (1+\rho+\delta)^{-1}[f(w) + \lambda_E\int_{V_N(w)}^T (v - f(w))dF_V(v|w) + \delta V_U]$$

and note that T_N is a fixed point of this mapping. Since $\mathcal{T}[T](w) < T$, we must have $T_N(w) < T$. ∎

B.5.1 Posted Fee Contracts

In Stevens (2004), the optimal contract is for the firm to "sell" itself to the worker for some fee, p, and allow the worker to receive the full value of the match. In this setup, the worker receives value $T - p$. We call these firms S firms, and we first establish that this wage-setting strategy strictly dominates the posted, non-negotiable wage strategy N. To see this, consider the profit from matching with a worker at an offered wage, w. Fixing the wage, there is an equivalent fee p, such that $V_N(w) = T - p$. While the worker is indifferent, the firm will prefer to offer the fee contract if $p > \Pi_N(w)$ which, since $p = T - V_N(w)$, will hold if $T > \Pi_N(w) + V_N(w) = T_N(w)$, which is in turn implied by the above lemma. Thus, in any equilibrium we must have $p_N = 0$. Next, observe that with only R and S firms in the market, it is impossible to profitably hire employed workers. While R firms will derive a profit of $(1-\alpha)(T - V_U)$ when they meet an unemployed worker, S firms will optimally post a fee contract of price $p^* = T - V_U$, which is acceptable to unemployed workers, and strictly preferred by the firm. Thus, the only equilibrium is $p_S = 1$.

B.5.2 Posted Wage with Renegotiation

We now consider the case in wage firms are able to post an *initially* non-negotiable wage (behave as if an N firm), with the option to later renegotiate (behave as an R firm). Here we assume that renegotiation occurs whenever the worker receives a wage offer that is preferred to their current arrangement, or when the worker encounters an R firm. We call these firms N^* firms. We start as before by showing that, conditional on a successful hire, firms strictly prefer to be N^* firms rather than N firms. Consider a successful hire by an N firm at wage w and suppose a q offer such that $V_{N^*}(q) = V_N(w)$. We will show that $\Pi_{N^*}(q) > \Pi_N(w)$. To see this note that once a worker has been successfully hired by an N^* firm the problem is equivalent to that of an R firm and so the joint value of the match is T. This gives $\Pi_{N^*}(q) = T - V_N(w)$. By comparison, we know that $\Pi_N(w) = T_N(w) - V_N(w)$ and since $T_N(w) < T$ we have the result, implying that $p_N = 0$. Once again, with only N^* and R firms in the market, it is only possible to profitably hire unemployed workers. For an R firm, the profit from doing so is $(1-\alpha)(T - V_U)$ while an N^* firm will optimally post an initial wage w^* such that $V_N(w^*) = V_U$ and enjoy profit $T - V_U$. Thus, the only equilibrium in this case is $p_{N^*} = 1$.

C Details on the Model with Heterogeneity

C.1 Steady State Characterization

We make use of the definition of the distribution of maximum attainable wages from the main text of the paper:

$$M(w) = p_R F_\theta(w) + (1 - p_R)\Phi(w).$$

We begin with the distribution of workers over this state, G. Flows out are equal to $(1-U)G(w)(\delta + \lambda_E \tilde{M}(w))$ while flows in are equal to $\lambda_E U(M(w) - \tilde{F}_\theta(w^*)$. Balancing flows gives

$$G(x) = \frac{M(x) - F_\theta(w^*)}{\tilde{F}_\theta(w^*)(1 + \kappa \tilde{M}(x))}.$$

Similarly, let g_N and g_R be the densities of employed workers at N and R firms over maximum attainable wages. Once again flows can be balanced to get:

$$g_N(x) = \frac{(1-p_R)(1+\kappa\tilde{F}_\theta(w^*))\tilde{F}(x)\phi(x)}{\tilde{F}_\theta(w^*)(1+\kappa\tilde{M}(x))^2} \tag{25}$$

$$g_R(x) = \frac{p_R(1+\kappa\tilde{F}_\theta(w^*)f_\theta(x)}{\tilde{F}_\theta(w^*)(1+\kappa\tilde{M}(x))^2} \tag{26}$$

The steady state mass of workers at R firms can be calculated by integrating g_R.

Since the maximum attainable wage is the actual wage at N firms, g_N is sufficient for determining the steady state wage distribution. For R firms, we also need to know the distribution of previous outside options. At all R firms with productivity x, the outflow of workers with outside option less than or equal to q is $(1-U)g_R(x)(\delta+\lambda_E\tilde{M}(q))H(q|x)$ while the flows in are $(\lambda_U U + \lambda_E(1-U)G(q))p_R f_\theta(x)$. Balancing flows results in:

$$H(q|x) = \left(\frac{1+\kappa\tilde{M}(x)}{1+\kappa\tilde{M}(q)}\right)^2$$

C.2 Proof of Proposition 3

We start by writing the expected profit to a firm with productivity θ making a wage offer w. Defining $\mu = \lambda_E/\lambda_U$, the probability that the firm meets an unemployed worker is $\frac{U}{U+\mu(1-U)} = \frac{\delta}{\delta+\lambda_E\tilde{F}_\theta(w^*)}$. If the worker is employed, the probability that the offer is acceptable to them is given by $G(w)$. Putting this together we get the probability of a successful hire:

$$\frac{\delta}{\delta+\lambda_E\tilde{F}_\theta(w^*)} + \frac{\lambda_E\tilde{F}_\theta(w^*)}{\delta+\lambda_E\tilde{F}_\theta(w^*)}G(w) = \frac{1}{1+\kappa M(w)}$$

The exit rate of this worker from the firm is given by $\delta+\lambda_E\tilde{M}(w)$ and so we get the expected profit:

$$\Pi_N(\theta,w) = \frac{1}{1+\kappa\tilde{M}(w)}\frac{\theta-w}{\rho+\delta+\lambda_E\tilde{M}(w)}.$$

In the following set of results, we make extensive use of this expression, which we simplify as:

$$\Pi_N(\theta,w) = \Gamma(w)(\theta-w).$$

Secondly, we assume the following tie-breaking rule: when two-firms make equally valuable wage offers, the worker moves from the incumbent to the new firm. This is an inconsequential assumption since such ties occur with zero probability. Assuming the alternative tie-breaking rule produces an observationally equivalent equilibrium outcome.

Lemma 1. *Let w^* be the reservation wage defined by $T(w^*) = U$. Then $w^* = \inf\{w : \Phi(w) > 0\}$.*

Proof. Define $\underline{w} = \inf\{w : \Phi(w) > 0\}$. Since all offers $w < w^*$ are, by definition, never accepted, we know that $\underline{w} \geq w^*$. Now assume that $\underline{w} > w^*$, and consider the optimal offer made by a firm when a match $x \in (w^*, \underline{w})$ is drawn. Since any offer $w \in (w^*, x)$ is both profitable to the firm and acceptable to an unemployed worker (who is met with positive probability), we have a contradiction. ∎

Lemma 2. Γ *is (i) strictly increasing; and (ii) continuous if and only if* Φ *is continuous.*

Proof. **(i)**: This follows directly from our assumption that F_θ is strictly increasing in w (the support of F_θ is a connected set) and Φ is, by definition, non-decreasing. Thus, Γ must be strictly increasing in w.

(ii): This is immediate, since Γ is a continuous transformation of Φ and F_θ. ∎

Lemma 3. *In equilibrium, the wage offer distribution Φ is continuous.*

Proof. Note that a discontinuity in Φ at some w implies a mass point at w and, by Lemma 2, Γ is discontinuous. Given the tie-breaking rule, we have that $\lim^+ \Gamma(w) > \Gamma(w)$. This is caused by a discontinuous increase in the probability of retaining a worker.[24] Hence, $\lim^+ \Pi_N(\theta, w) > \Pi_N(\theta, w)$ for any θ, and for any firm offering wage w, an improvement in profit can be made by offering $w + \epsilon$ where ϵ is arbitrarily small. Thus no firm prefers to offer w, a contradiction. ∎

The following corollary is immediate.

Corollary 1. Γ *is continuous.*

Lemma 4. *In equilibrium, wages are given by an almost everywhere deterministic function, φ.*

Proof. Suppose otherwise. Then for a firm with match θ, the firm is indifferent over a set $\mathcal{W}$ with positive Lebesgue measure:

$$\Gamma(w)(\theta - w) = c, \ \forall \ w \in \mathcal{W}.$$

Likewise, for a firm with match $\hat{\theta} \neq \theta$, indifference is achieved over a set $\hat{\mathcal{W}}$:

$$\Gamma(w)(\hat{\theta} - w) = \hat{c}, \ \forall \ w \in \hat{\mathcal{W}}.$$

If $\mathcal{W} \cap \hat{\mathcal{W}}$ has positive measure, we must have $\Gamma(w)(\theta - \hat{\theta}) = c - \hat{c}$ for all w in this intersection, which can be true only if $\Gamma(w)$ is everywhere constant, contradicting Lemma 2. Therefore, $\mathcal{W} \cap \hat{\mathcal{W}} = \emptyset$, and so this can only be true for a countable set of matches, which have measure zero under our regularity assumptions on F_θ.

∎

Lemma 5. *The wage offer function, φ, is strictly increasing in match values, θ.*

Proof. Let $\varphi(\theta) = w$. This implies that:

$$\Gamma(w)(\theta - w) > \Gamma(\hat{w})(\theta - \hat{w}), \ \forall \hat{w} < w$$

Rearranging this expression we get:

$$(\Gamma(w) - \Gamma(\hat{w}))\theta > \Gamma(w)w - \Gamma(\hat{w})\hat{w}, \ \forall \hat{w} < w$$

By Lemma 2, $\Gamma(w) - \Gamma(\hat{w}) > 0$, which implies that for any $\theta' > \theta$, we have

$$(\Gamma(w) - \Gamma(\hat{w}))\theta' > \Gamma(w)w - \Gamma(\hat{w})\hat{w}, \ \forall \hat{w} < w$$

So when the match value is θ', the above inequality implies that w is also preferred to all $\hat{w} < w$, and so $\varphi(\theta') \geq \varphi(\theta)$, However, if this inequality is not strict, repeated application of the above inequality implies that $\varphi(z) = w$ for all $z \in [\theta, \theta']$. However, this implies a discontinuity in Φ, contradicting Lemma 3. Thus, the inequality must be strict. ∎

To prove differentiability, we make use of the following commonly known result.

Lemma 6. *If a function, $f : \mathbb{R} \mapsto \mathbb{R}$, is bounded, and monotonically increasing, it is almost everywhere (according to Lebesgue measure) differentiable.*

[24] Notice that if we had assumed the alternative tie-breaking rule, there would be a discontinuous increase in the probability of hiring the worker, and the result would still follow.

Proof. See, for example, Result 11.42 in Titchmarsh (1932). ∎

Lemma 7. *The wage offer function, φ is almost everywhere differentiable and lower semi-continuous.*

Proof. Consider the function φ on the domain $[w^*, \theta]$. Since $\varphi(\theta)$ is bounded above by θ, bounded below by w^*, and strictly monotonically increasing, it follows from Lemma 6 that φ must be almost everywhere differentiable (and hence almost everywhere continuous). Consider now a potential discontinuity in φ at θ. Let d^+ and d^- denote the differentiation operation, taking *right* and *left* limits, respectively. Let $\varphi^-(\theta) = w_0$ and $\varphi^+(\theta) = w_1$. We know that $w_0 < w_1$. A discontinuity in φ implies that the distribution Φ is flat over the range $[w_0, w_1]$, and hence: $d^+\Phi(w_0) = d^-\Phi(w_1) = 0$. Suppose, for contradiction, that the function is upper semicontinuous, such that $\varphi(\theta) = w_1$. Optimality of this wage choice implies that the pair of inequalities

$$d^+\Pi_N(\theta, w_1) \leq 0, \qquad d^-\Pi_N(\theta, w_1) \geq 0$$

must hold. Taking left and right derivatives at this point gives inequalities

$$\begin{aligned}\frac{\lambda_E(\rho + 2(\delta + \lambda_E\tilde{M}(w_1)))(p_R f_\theta(w_1) + (1-p_R)d^+\Phi(w_1))}{(\rho+\delta+\lambda_E\tilde{M}(w_1))(\delta+\lambda_E\tilde{M}(w_1))(\theta - w_1)} - 1 \leq 0 \\ \frac{\lambda_E(\rho + 2(\delta + \lambda_E\tilde{M}(w_1)))(p_R f_\theta(w_1) + 0)}{(\rho+\delta+\lambda_E\tilde{M}(w_1))(\delta+\lambda_E\tilde{M}(w_1))(\theta - w_1)} - 1 \geq 0\end{aligned}$$

Since $d^+\Phi(w_1) = \phi(w_1) > 0$, one inequality here contradicts the other. Hence, φ must be lower semi continuous (application of the necessary inequalities at w_0 yields no such contradiction). ∎

C.3 Solving Wages at R Firms

The value of being at an R firm, given a previous outside option of q is:

$$\begin{aligned}(\rho+\delta+\lambda_E\tilde{M}(x))V_R(x,q) &= \varphi_R(x,q) + \lambda_E(1-\alpha)\int_q^x (T(y) - T(q))dM \\ &\quad + \lambda p_R \int_x [(1-\alpha)T(x) + \alpha T(y)]dF + \lambda(1-p_R)\int_x T(y)d\Phi \qquad (27)\end{aligned}$$

Using the identity: $\int_a^b TdG = \int T'\tilde{G}dx + \tilde{G}(a)T(a) - \tilde{G}(b)T(a)$ we get:

$$\begin{aligned}(\rho+\delta+\lambda_E\tilde{M}(x))V_R(x,q) &= \varphi_R(x,q) + \lambda_E(1-\alpha)\int_q^x S'(y)\tilde{M}(y)dy + \lambda_E p_R\alpha\int_x T'(y)\tilde{F}(y)dy \\ &+ \lambda_E(1-p_R)\int_x T'(y)\tilde{\Phi}(y)dy + \lambda_E(1-\alpha)\tilde{M}(x)(T(q)-T(x)) + \underbrace{\lambda_E p_R\tilde{F}(x)T(x) + \lambda(1-p_R)\tilde{\Phi}(x)T(x)}_{=\lambda\tilde{M}(x)T(x)} \qquad (28)\end{aligned}$$

Which simplifies to

$$(\rho+\delta)V_R(x,q) = \varphi_R(x,q) + \lambda_E(1-\alpha)\int_q^x T'(y)\tilde{M}(y)dy + \lambda_E p_R\alpha\int_x T'(y)\tilde{F}(y)dy + \lambda_E(1-p_R)\int_x T'(y)\tilde{\Phi}(y)dy$$

Similarly, we get:

$$\begin{aligned}(\rho+\delta)T(x) &= x + \lambda_E p_R\alpha\int_x (T(y)-T(x))dF + \lambda_E(1-p_R)\int_x (T(y)-T(x))d\Phi(x) \qquad (29) \\ &= x + \lambda_E p_R\alpha\int_x T'(y)\tilde{F}(y)dy + \lambda_E(1-p_R)\int_x T'(y)\tilde{\Phi}(y)dy \qquad (30)\end{aligned}$$

Rearranging (29) and differentiating gives:

$$T'(x) = \frac{1}{\rho + \delta + \lambda_E \alpha \tilde{M}(x)}$$

and imposing that $V_R(x,q) = T(q) + \alpha(T(x) - T(q))$ gives the wage equation:

$$\varphi_R(x,q) = \alpha x + (1-\alpha)q - \lambda_E p_R (1-\alpha)^2 \int_q^x \frac{\tilde{F}(y)}{\delta + \lambda_E p_R \alpha \tilde{F}(y) + \lambda_E (1-p_R)\tilde{\tilde{\Phi}}(y)} dy.$$

C.3.1 Derivation of an Additive Log Wage Equation

Following Bagger et al. (2014), assume that workers derive utility from log wages, which are determined through log piece-rates r, yielding a flow utility $a + \theta + z + r$ where a is log worker ability, θ is log match productivity, and z is log firm productivity. We additionally assume that with probability $1 - p_R$ the piece rate offer is drawn from a posting firm. We let Φ now indicate the resulting distribution of $r + \theta + z$ for these firms. We leave the problem of solving for this piece rate distribution in equillibrium to future research. When matched with an R-firm, bargaining delivers a piece rate $r(q,\theta)$ that solves

$$V(a, r(q,\theta), \theta) = V(a, 0, q) + \alpha[V(a,0,\theta) - V(a,0,q)].$$

From here, algebra proceeds identically to the baseline model. Assuming that $b(a) = b + a$ one can first show that all value functions are additively separable in log ability, a. Next, one can derive expressions for $V(0,\theta)$ that are identical to the equations that define $T(\theta)$. This delivers an identical algebraic derivation of the piece rate:

$$r(q,x) = -\left[(1-\alpha)(x-q) + \lambda_E p_R (1-\alpha)^2 \int_q^x \frac{\tilde{F}_{z+\theta}(y|R)}{\rho + \delta + p_R \alpha \tilde{F}_{z+\theta}(y|R) + (1-p_R)\tilde{\tilde{\Phi}}(y)} dy\right]$$

where x is the total log productivity of the match, $F_{\theta+z}(\cdot|R)$ is the CDF of total log productivity, and q is the maximum attainable log wage at the outside option. The piece rate can be equivalently written as

$$-(1-\alpha)\int_q^x \frac{\rho + \delta + p_R \tilde{F}_{z+\theta}(y|R) + (1-p_R)\tilde{\tilde{\Phi}}(y)}{\rho + \delta + p_R \alpha \tilde{F}_{z+\theta}(y|R) + (1-p_R)\tilde{\tilde{\Phi}}(y)} dy$$

Combining all terms of the log wage (w) for worker i at firm j yields a decomposition:

$$w_{ijt} = a_i + z_j + \theta_{ij} + R_j r(q_{it}, z_j + \theta_{ij}) + (1-R_j) r_j$$

where R_j indicates whether firm j is a negotiator, and q_{it} is the most recent maximum attainable log wage for worker i at time t.

C.4 Solving Wages at N Firms

Applying the steady state rate of unemployment and distribution of workers over maximum attainable wages (G), the probability of a hire is

$$\frac{1}{1 + \kappa \tilde{M}(w)}$$

and so the expected present value of an offer w is:

$$\Pi_N(\theta, w) = \frac{1}{1 + \kappa \tilde{M}(x)} \frac{\theta - w}{\rho + \delta + \lambda_E \tilde{M}(w)}.$$

For the points at which the first order condition holds, we use $\Phi(w) = F(\varphi^{-1}(w))$ to characterize the equilibrium offer function using the first order condition:

$$(p_R f(\varphi(\theta)) + (1-p_R) f(\theta)/\varphi'(\theta))\,(\theta - \varphi(\theta)) = \frac{\left(\delta + \lambda_E p_R \tilde{F}(\varphi(\theta)) + \lambda_E(1-p_R)\tilde{F}(\theta)\right)\left(\rho + \delta + \lambda_E p_R \tilde{F}(\varphi(\theta)) + \lambda_E(1-p_R)\tilde{F}(\theta)\right)}{\left(\rho + 2(\delta + \lambda_E p_R \tilde{F}(\varphi(\theta)) + \lambda_E(1-p_R)\tilde{F}(\theta))\right)}$$

which can be rearranged into the ODE shown in the text.

One issue in using the differential equation above is that Proposition 3 does not guarantee that φ is everywhere continuous, and the first-order condition is known only to be necessary and not sufficient. The algorithm we use accounts for potential discontinuities in φ by globally checking for optimality at each step.

To do this, we need to use the following profit function, which gives the profit to the firm under the equilibrium condition that wage offers are ranked according to θ.

$$\Pi^*(\theta, w) = \frac{\theta - w}{(\delta + \lambda_E p_R \tilde{F}_\theta(w) + \lambda_E(1-p_R)\tilde{F}_\theta(\theta))(\rho + \lambda_E p_R \tilde{F}_\theta(w) + \lambda_E(1-p_R)\tilde{F}_\theta(\theta))} \tag{31}$$

The algorithm proceeds as follows, given a predetermined grid $\{\theta_0, \theta_1, ..., \theta_J\}$ with $\theta_0 = \theta^*$. To initialize the algorithm, we set $w_0 = \theta^*$:

1. Given θ_{j-1}, $w_{j-1}(= \varphi(\theta_{j-1}))$, use the ODE and either Euler's method or a more advanced method such as Runge-Kutta to compute the step $d\varphi_j$.

2. Check for global optimality by solving $w^* = \arg\max_{w \in [w_{j-1}, \theta_j]} \Pi^*(\theta_j, w)$.

3. If $w^* > w_{j-1}$, set $w_{j+1} = w^*$.

4. Otherwise, set $w_j = w_{j-1} + d\varphi_j$.

The idea here is that, if $w^* > w_{j-1}$, then the shape of the match distribution F supports a discontinuity at θ_j, such that no firm offers between w_{j-1} and w^*. In addition, the marginal firm θ_j is indifferent between these wage offers. If, on the other hand, the firm prefers to offer w_j (the lowest wage available) then we must introduce marginal wage competition by way of $\varphi'(\theta)$.

C.5 Firm Profits

Expected profit for an R firm with match θ can be written as:

$$\begin{aligned}
\Pi_R(\theta) &= \frac{(1-\alpha)}{\delta + \lambda_E \tilde{M}(w^*)}\left[\delta(T(\theta) - T(w^*)) + \lambda_E \tilde{M}(w^*)\int_{w^*}^{\theta}(T(\theta) - T(y))dG\right] \\
&= \frac{(1-\alpha)}{\delta + \lambda_E \tilde{M}(w^*)}\left[\delta(T(\theta) - T(w^*)) + \lambda_E \tilde{M}(w^*)\int_{w^*}^{\theta} T'(y)G(y)dy\right] \\
&= \frac{(1-\alpha)\delta}{\delta + \lambda_E \tilde{M}(w^*)}\int_{w^*}^{\theta} T'(y)\left[1 + \frac{\lambda_E M(y)}{\delta + \lambda_E \tilde{M}(y)}\right]dy \\
&= (1-\alpha)\delta\int_{w^*}^{\theta}\frac{T'(y)}{\delta + \lambda_E \tilde{M}(y)}dy
\end{aligned}$$

This gives us:

$$\Pi_R'(\theta) = (1-\alpha)\frac{1}{\delta}\frac{1}{(1 + \kappa\alpha p_R \tilde{F}_\theta(\theta) + \kappa(1-p_R)\tilde{\Phi}(\theta))(1 + \kappa\tilde{M}(\theta))}$$

Integrating over the distribution of potential match values, F_θ, gives:

$$\Pi_R = \int_{w^*} \Pi_R(y) dF_\theta(y)$$

which we can evaluate using integration by parts:

$$\Pi_R = \frac{1}{\delta}(1-\alpha)\int_{w^*}^{\tilde{\theta}} \frac{\tilde{F}_\theta(y)}{(1+\kappa\alpha p_R \tilde{F}_\theta(y) + \kappa(1-p_R)\tilde{\Phi}(y))(1+\kappa\tilde{M}(y))} dy.$$

Similarly, expected profit for N firms can be written as:

$$\Pi_N = \int_{w^*} \Pi_N(y, \varphi(y)) dF(y)$$

Once again integration by parts gives:

$$\Pi_N = \int_{w^*} \frac{d}{d\theta}\Pi_N(y, \varphi(y))\tilde{F}_\theta(y) dy.$$

Now, assuming that the first order condition holds at each point in $\varphi(\theta)$, the envelope theorem delivers:

$$\frac{d}{d\theta}\Pi(y, \varphi(y)) = \frac{1}{(1+\kappa\tilde{M}(\varphi(y)))^2}$$

and so we get:

$$\Pi_N = \frac{1}{\delta}\int_{w^*} \frac{\tilde{F}_\theta(y)}{(1+\kappa\tilde{M}(\varphi(y)))^2} dy$$

C.6 Reservation Wage

The value of unemployment is:

$$\rho U = b + \lambda_0\left(\alpha p_R \int_{w^*}(T(x) - T(w^*))dF(x) + (1-p_R)\int_{w^*}(T(x) - T(w^*))d\Phi(x)\right)$$

Setting $T(w^*) = U$, the wage at which the total value of the match is equal to unemployment, gives the reservation wage equation:

$$w^* = b + (\lambda_0 - \lambda_1)\left(\alpha p_R \int_{w^*}(T(x) - T(w^*))dF(x) + (1-p_R)\int_{w^*}(T(x) - T(w^*))d\Phi(x)\right)$$

Which can be written as:

$$w^* = b + (\lambda_U - \lambda_E)\int_{w^*} \frac{\alpha p_R \tilde{F}_\theta(x) + (1-p_R)\tilde{\Phi}(x)}{\rho + \delta + \lambda_E \alpha p_R \tilde{F}_\theta(x) + \lambda_E(1-p_R)\tilde{\Phi}(x)} dx$$

D Tables

Table 5: Moments Used for Inner Loop of Minimum Distance Estimator ($m_2(x)$)

			EU	EE	U
<High School	20-29	Male	0.05	0.01	0.16
<High School	20-29	Female	0.05	0.02	0.19
<High School	30-39	Male	0.04	0.01	0.09
<High School	30-39	Female	0.03	0.01	0.12
<High School	40-49	Male	0.04	0.01	0.09
<High School	40-49	Female	0.03	0.01	0.08
<High School	50-59	Male	0.03	0.01	0.07
<High School	50-59	Female	0.03	0.01	0.07
High School	20-29	Male	0.03	0.02	0.09
High School	20-29	Female	0.03	0.02	0.07
High School	30-39	Male	0.02	0.01	0.05
High School	30-39	Female	0.02	0.01	0.05
High School	40-49	Male	0.02	0.01	0.05
High School	40-49	Female	0.02	0.01	0.04
High School	50-59	Male	0.02	0.01	0.04
High School	50-59	Female	0.02	0.01	0.04
Bachelor's	20-29	Male	0.02	0.02	0.04
Bachelor's	20-29	Female	0.02	0.02	0.03
Bachelor's	30-39	Male	0.02	0.01	0.02
Bachelor's	30-39	Female	0.02	0.01	0.03
Bachelor's	40-49	Male	0.01	0.01	0.02
Bachelor's	40-49	Female	0.02	0.01	0.02
Bachelor's	50-59	Male	0.02	0.01	0.03
Bachelor's	50-59	Female	0.02	0.01	0.03
>Bachelor's	20-29	Male	0.01	0.01	0.04
>Bachelor's	20-29	Female	0.02	0.02	0.03
>Bachelor's	30-39	Male	0.02	0.01	0.02
>Bachelor's	30-39	Female	0.02	0.01	0.02
>Bachelor's	40-49	Male	0.01	0.01	0.01
>Bachelor's	40-49	Female	0.02	0.01	0.02
>Bachelor's	50-59	Male	0.02	0.01	0.02
>Bachelor's	50-59	Female	0.02	0.01	0.02

Source: CPS 2008 Monthly Files (Flood et al., 2020)

Table 6: Moments Used for Outer Loop of Minimum Distance Criterion ($m_1(x)$) and Model Fit

			$B(x)$		Res Wage Eqn		$\mathbb{E}[\log(W_{ss})\|x] - \mathbb{E}[\log(W_{ue})\|x]$			$\mathbb{V}[\log(W_{ss})\|x] - \mathbb{V}[\log(W_{ue})\|x]$		
			Model	Data	Model	Data	Model	Data	St Dev	Model	Data	St Dev
<High School	20-29	Male	0.17	0.17	-0.01	0.0	0.11	0.14	0.03	-0.01	-0.01	0.05
<High School	20-29	Female	0.0	0.0	-0.0	0.0	0.13	0.14	0.04	0.0	-0.03	0.06
<High School	30-39	Male	0.49	0.5	0.01	0.0	0.06	0.02	0.05	0.01	-0.02	0.05
<High School	30-39	Female	0.17	0.18	0.01	0.0	0.09	0.04	0.06	-0.0	0.01	0.13
<High School	40-49	Male	0.22	0.21	-0.01	0.0	0.07	0.12	0.05	0.0	0.06	0.03
<High School	40-49	Female	0.3	0.29	0.01	0.0	0.05	0.03	0.06	-0.0	0.11	0.08
<High School	50-59	Male	0.37	0.36	-0.02	0.0	0.04	0.12	0.06	0.0	0.15	0.04
<High School	50-59	Female	0.17	0.17	-0.0	0.0	0.04	0.06	0.1	-0.0	-0.01	0.12
High School	20-29	Male	0.36	0.37	0.05	0.0	0.22	0.12	0.02	0.0	0.01	0.03
High School	20-29	Female	0.2	0.21	0.01	0.0	0.2	0.12	0.02	-0.01	0.05	0.02
High School	30-39	Male	0.33	0.33	-0.01	0.0	0.13	0.22	0.03	0.01	0.02	0.03
High School	30-39	Female	0.31	0.32	-0.0	0.0	0.14	0.22	0.04	0.01	-0.08	0.08
High School	40-49	Male	0.45	0.45	-0.01	0.0	0.11	0.22	0.03	0.01	0.02	0.03
High School	40-49	Female	0.29	0.3	-0.01	0.0	0.1	0.18	0.04	0.01	-0.08	0.1
High School	50-59	Male	0.32	0.33	-0.01	0.0	0.11	0.22	0.04	0.01	0.03	0.02
High School	50-59	Female	0.33	0.31	-0.01	0.0	0.08	0.24	0.04	0.01	0.04	0.1
Bachelor's	20-29	Male	0.52	0.52	0.0	0.0	0.24	0.23	0.07	0.02	-0.03	0.07
Bachelor's	20-29	Female	0.34	0.34	0.06	0.0	0.32	0.18	0.07	0.0	-0.24	0.21
Bachelor's	30-39	Male	0.56	0.55	-0.01	0.0	0.14	0.23	0.07	0.01	0.01	0.05
Bachelor's	30-39	Female	0.35	0.36	0.01	0.0	0.21	0.13	0.07	0.01	-0.02	0.09
Bachelor's	40-49	Male	0.46	0.45	-0.02	0.0	0.11	0.47	0.11	0.01	0.05	0.07
Bachelor's	40-49	Female	0.33	0.3	-0.02	0.0	0.13	0.34	0.08	0.01	0.15	0.04
Bachelor's	50-59	Male	0.51	0.51	-0.01	0.0	0.15	0.35	0.1	0.02	-0.05	0.07
Bachelor's	50-59	Female	0.51	0.51	-0.01	0.0	0.11	0.32	0.08	0.01	0.12	0.07
>Bachelor's	20-29	Male	0.78	0.8	-0.01	0.0	0.21	0.13	0.21	0.02	-0.08	0.18
>Bachelor's	20-29	Female	0.74	0.75	-0.03	0.0	0.2	0.28	0.14	0.02	0.11	0.08
>Bachelor's	30-39	Male	0.67	0.64	-0.06	0.0	0.13	0.77	0.16	0.01	0.13	0.07
>Bachelor's	30-39	Female	0.51	0.53	0.01	0.0	0.11	-0.06	0.12	0.01	-0.06	0.11
>Bachelor's	40-49	Male	0.57	0.58	-0.0	0.0	0.07	0.11	0.15	0.01	-0.1	0.13
>Bachelor's	40-49	Female	0.48	0.44	-0.02	0.0	0.18	0.5	0.09	0.01	0.11	0.06
>Bachelor's	50-59	Male	0.54	0.56	0.03	0.0	0.09	-0.12	0.3	0.01	-0.59	0.54
>Bachelor's	50-59	Female	0.37	0.41	0.0	0.0	0.1	-0.1	0.2	0.0	-0.5	0.29

Source: Wage data taken from CPS 2008 Monthly and ORG Files (Flood et al., 2020). Bargaining data taken from Hall and Krueger (2012)

Table 7: Estimates of Group-Specific Parameters

			λ_U	λ_E	δ	α
<High School	20-29	Male	0.29	0.04	0.05	0.36
			(0.02)	(0.01)	(0.0)	(0.03)
<High School	20-29	Female	0.21	0.05	0.05	0.41
			(0.02)	(0.01)	(0.0)	(0.03)
<High School	30-39	Male	0.62	0.06	0.04	0.42
			(0.14)	(0.02)	(0.0)	(0.03)
<High School	30-39	Female	0.28	0.05	0.03	0.43
			(0.04)	(0.01)	(0.0)	(0.04)
<High School	40-49	Male	0.56	0.05	0.04	0.38
			(0.09)	(0.01)	(0.0)	(0.03)
<High School	40-49	Female	0.6	0.03	0.03	0.34
			(0.18)	(0.01)	(0.0)	(0.04)
<High School	50-59	Male	0.71	0.03	0.03	0.35
			(0.18)	(0.02)	(0.0)	(0.05)
<High School	50-59	Female	0.61	0.03	0.03	0.33
			(0.19)	(0.01)	(0.0)	(0.05)
High School	20-29	Male	0.34	0.06	0.03	0.53
			(0.01)	(0.0)	(0.0)	(0.01)
High School	20-29	Female	0.38	0.06	0.03	0.52
			(0.01)	(0.0)	(0.0)	(0.01)
High School	30-39	Male	0.44	0.04	0.02	0.48
			(0.02)	(0.0)	(0.0)	(0.01)
High School	30-39	Female	0.44	0.04	0.02	0.49
			(0.02)	(0.0)	(0.0)	(0.01)
High School	40-49	Male	0.54	0.03	0.02	0.47
			(0.02)	(0.0)	(0.0)	(0.01)
High School	40-49	Female	0.61	0.04	0.02	0.46
			(0.03)	(0.0)	(0.0)	(0.01)
High School	50-59	Male	0.55	0.04	0.02	0.47
			(0.03)	(0.0)	(0.0)	(0.01)
High School	50-59	Female	0.74	0.04	0.02	0.44
			(0.04)	(0.0)	(0.0)	(0.01)
Bachelor's	20-29	Male	0.57	0.05	0.02	0.56
			(0.05)	(0.01)	(0.0)	(0.02)
Bachelor's	20-29	Female	0.52	0.08	0.02	0.59
			(0.05)	(0.01)	(0.0)	(0.01)
Bachelor's	30-39	Male	0.88	0.05	0.02	0.54
			(0.18)	(0.01)	(0.0)	(0.02)
Bachelor's	30-39	Female	0.63	0.05	0.02	0.55
			(0.07)	(0.01)	(0.0)	(0.02)
Bachelor's	40-49	Male	1.04	0.05	0.01	0.53
			(0.26)	(0.01)	(0.0)	(0.02)
Bachelor's	40-49	Female	0.8	0.04	0.02	0.5
			(0.11)	(0.01)	(0.0)	(0.02)
Bachelor's	50-59	Male	0.58	0.03	0.02	0.53
			(0.08)	(0.0)	(0.0)	(0.02)
Bachelor's	50-59	Female	0.8	0.03	0.02	0.47
			(0.09)	(0.0)	(0.0)	(0.02)
>Bachelor's	20-29	Male	0.4	0.03	0.01	0.58
			(0.09)	(0.02)	(0.0)	(0.06)
>Bachelor's	20-29	Female	0.65	0.05	0.02	0.59
			(0.12)	(0.02)	(0.0)	(0.03)
>Bachelor's	30-39	Male	1.15	0.05	0.02	0.57
			(0.2)	(0.01)	(0.0)	(0.02)
>Bachelor's	30-39	Female	0.88	0.04	0.02	0.54
			(0.12)	(0.01)	(0.0)	(0.02)
>Bachelor's	40-49	Male	1.51	0.05	0.01	0.52
			(0.98)	(0.03)	(0.0)	(0.02)
>Bachelor's	40-49	Female	0.68	0.05	0.02	0.58
			(0.07)	(0.01)	(0.0)	(0.02)
>Bachelor's	50-59	Male	0.82	0.03	0.02	0.51
			(0.09)	(0.01)	(0.0)	(0.03)
>Bachelor's	50-59	Female	0.95	0.04	0.02	0.53
			(0.12)	(0.01)	(0.0)	(0.02)

See main text for description of parameters and estimation procedure. Standard errors calculated using 100 bootstrap samples.

Table 8: The Contribution of Differences in Wage-Setting to Wage Inequality

Statistic (% pop val)	Education	$p_R = 0$	$p_R = 1$
Within-Group Variance	<High School	-0.74	2.27
		(0.33)	(0.74)
Within-Group Variance	High School	-1.22	-1.43
		(0.07)	(0.21)
Within-Group Variance	Bachelor's	-1.46	-2.4
		(0.14)	(0.52)
Within-Group Variance	>Bachelor's	-1.03	-1.04
		(0.16)	(0.37)
Gender Wage Gap	<High School	-6.59	-9.9
		(4.74)	(5.35)
Gender Wage Gap	High School	-6.82	3.76
		(1.51)	(2.24)
Gender Wage Gap	Bachelor's	-8.98	-15.15
		(3.53)	(4.78)
Gender Wage Gap	>Bachelor's	-6.81	-12.97
		(2.26)	(4.2)

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