Student Name:
Student ID:
Section Number:

ECON 4261: Midterm

Thursday Feb 23, 2021

Good Luck!

Instructions

- **Write your name, student ID, and section number** on this page above **and** on your answer booklets.
- Write all your answers in the booklet provided.
- Please ensure that your answers are **neat** and **legible**.
- Please place this exam inside your answer booklet before you submit it.

Setup for all questions

Suppose that you have an iid sample: $(W_n, F_n)_{n=1}^N$ where W_n is person n's hourly wage and F_n is a binary variable equal to 1 if person n is female and 0 otherwise.

Consider the linear model

$$\mathbb{E}[\log(W_n)|F_n] = \beta_0 + \beta_1 F_n.$$

(a) Consider the null hypothesis that $\mathbb{E}[\log(W_n)|F_n = 1] = \mathbb{E}[\log(W_n)|F_n = 0]$. Show that this is equivalent to $\beta_1 = 0$. (2 points)

(b) Carefully describe how you would estimate the parameters β. You can be concise, just describe how you would get $\hat{\beta}$ from the data in a way that someone else could replicate. **Do not** just say what command you would use in the `R` programming language. (2 points)

(c) What is the expected value of $\hat{\beta}$? (1 point)

(d) What is the probability limit of $\hat{\beta}$ as $N \to \infty$? Which theorem guarantees this? (2 points)

(e) What is the asymptotic distribution of $\hat{\beta}$? Which theorem guarantees this? (2 points)

(f) Carefully describe a two-sided, size α test of the null hypothesis from part (a) using your data. Again, you can be concise, but you must provide enough details that someone else could replicate your approach. **Do not** just say what command you would use in the `R` programming language. (4 points)

(g) Suppose that:

$$\hat{\beta} \sim \mathcal{N}(\beta, \hat{V}_{\hat{\beta}}).$$

Consider the following confidence interval:

$$\widehat{CI} = [\hat{\beta}_1 - z_{\alpha/3} \times \sqrt{\hat{v}_1}, \hat{\beta}_1 + z_{2\alpha/3} \times \sqrt{\hat{v}_1}]$$

where $\hat{v}_1$ is taken from the second row and second column of the matrix $\hat{V}_{\hat{\beta}}$.

Show that this is a valid $(1-\alpha) \times 100\%$ confidence interval for $\hat{\beta}_1$. (4 points)

(h) Why would the the usual (symmetric) confidence interval be considered preferable to the one described in part (g)? (1 point)

(i) Recall that

$$\epsilon_n = \log(W_n) - \mathbb{E}[\log(W_n)|F_n].$$

Now suppose that

$$\epsilon_n = (1 - F_n)\epsilon_{n,0} + F_n\epsilon_{n,1}$$

where

$$\mathbb{E}[\epsilon_{n,0}^2] = \sigma^2, \ \mathbb{E}[\epsilon_{n,1}^2] = 2\sigma^2.$$

Use this to calculate $\mathbb{V}[\epsilon|F_n]$. Based on this, does the assumption of homoskedasticity hold in this case? (2 point)