Family Law Effects on Divorce, Fertility and Child Investment *

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Abstract

In order to assess the child welfare impact of policies governing divorced parenting, such as child support orders, child custody assignments, and marital dissolution standards, one must consider their influence not only on the divorce rate but also on spouses' fertility choices and child investments. We develop a model of marriage, fertility and parenting, with the main goal being the investigation of how policies toward divorce influence outcomes for husbands, wives and children. We estimate parental preferences and a child cognitive ability production function using data on parental time allocation decisions, fertility, and divorce outcomes. Using the model estimates we examine the effects of changes in divorce requirements and child support standards on outcomes for intact and divided families. Our simulations indicate that bilateral divorce standards and outright divorce bans increase fertility and substantially reduce divorce, but they have quite negative consequences for children's cognitive attainment. Our parameter estimates and motivating empirical evidence suggest that this surprising result arises through the damaging effect on children of prolonging poor quality marriages. On the other hand, while large increases in child support weakly increase divorce rates, they too lead to (here very small) declines in children's ability. Together, our simulations indicate that the divorce rate is a very poor summary statistic for the success of family law, as children's interests are not necessarily best served by attempting to minimize the divorce rate among parents.

1 Introduction

Divorced parenting in the U.S. is regulated through a combination of laws regarding marital dissolution, child custody and placement, and the assignment and enforcement of child

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support obligations. The primary objective of these rules is to increase the well-being of children and parents, and the divorce rate is often regarded as a first order measure of the success of family law. The rationale for this focus is the preponderance of empirical evidence that suggests that children living in households without both biological parents are more likely to suffer from behavioral problems and have lower levels of a broad range of achievement indicators measured at various points over the life cycle (see, e.g., Haveman and Wolfe 1995). Empirical studies of unilateral divorce laws and child support enforcement have isolated the effects of changes in such legal structures on divorce rates (e.g., Friedberg 1998, Gruber 2004, Wolfers 2006, and Nixon 1997). Recent empirical studies of gender-neutral custody standards and financial incentives for joint custody provide new evidence on the effects of custody policy reforms on children's attainment (Chen and Logan 2020 and Kranz et al. 2021). A complete picture of the influence of family law on family members' welfare should include an understanding of the mechanisms by which family law changes influence fertility, child outcomes, and the distribution of resources within the family, in addition to divorce rates. Our objective is to take a first step in this direction by modeling the interactions of married couples over fertility, child investment, and divorce in the shadow of existing divorce regulations. Our goal is to understand, and quantitatively evaluate, the weight of the law in shaping parental behavior and, ultimately, child outcomes.[1]

Most studies of parental decision-making regarding investments in children and child outcomes take a very limited view of parental interactions. In many cases, the father is considered a passive agent in this process, whose role is limited to providing income for the household (as in Bernal 2008, Bernal and Keane 2011, Mullins 2019 and 2020, and Liu et al. 2010). In some cases, parents' preferences have been represented by a unitary utility function, as in Del Boca et al. (2014, 2022), which obscures the question of how these unitary preferences are formed from the individual preferences of the parents prior to or after marriage.[2] Much of what we have learned about parents' dynamic decision-making, therefore, has been in the context of a mother's (or mother and father's, assuming a unitary objective) individual dynamic optimization problem. When studying the influence of divorce law on the family, however, the distinct choices of mothers and fathers are paramount. For example, it is virtually impossible to understand the influence of potential child support on fertility, investment, and divorce decisions by studying the mother's perspective in isolation. Hence we model the choices of mothers and fathers as an ongoing, simultaneous-move game. Our model and data begin from the date of marriage, which, while excluding a substantial and non-random segment of parents, has the benefit of granting access to similar information on the mother and father when early fertility, investment

[1] Our more formal analysis builds on the original insights of Mnookin and Kornhauser (1979).

[2] A notable exception is Verriest (2022), who uses the framework of Del Boca et al. (2014) to examine investments in children and child outcomes in intact households. Unlike Del Boca et al., he allows parents to have different preferences over child outcomes and labor supply. Parents' decisions are coordinated through the maximization of a weighted average of their utilities.

and divorce choices are being made.

We draw on an extensive empirical literature on marriage dynamics, including Aiyagari, Greenwood and Guner (2000), Brien, Lillard and Stern (2006), Chiappori, Fortin and Lacroix (2002), Bruze, Svarer, and Weiss (2015), and Voena (2015). This literature emphasizes the repeated interaction of a husband and wife in deciding whether to continue a marriage and the allocation of household resources. With respect to this literature, our contribution is to endogenize fertility and child investment decisions both during and after marriage. Given that parents are forward-looking, divorce laws and regulations influence all of these decisions both within intact marriages and when parents are divorced.

We draw methods and insights from a comparatively recent line of research that models the investment decisions of households with respect to child "quality," which is taken to be cognitive ability. In particular, our framework shares certain modeling choices with those found in Del Boca, Flinn, and Wiswall (2014, 2016). These authors examine spouses' time allocation decisions, including investment time with the child and market labor supply, under the assumption of a Cobb-Douglas production technology forming the cognitive ability of the child and a Cobb-Douglas household utility function. Caucutt, Lochner, Mullins, and Park (2020) further expand the set of productive inputs to examine the balance among two parents' time inputs, market goods, and market child care in generating children's attainment. We introduce marriage dynamics and fertility into this framework, with the risk of marital dissolution and the laws governing divorced parenting allowed to influence the choices made by parents prior to and following divorce and even impacting the decision to have a child.

Two prior papers bear noting, as they also model marriage dynamics, fertility, and child investment. Caucutt, Guner and Knowles (2002) model marriage dynamics, fertility, and child expenditures. Their model is more comprehensive than ours on some dimensions, as it includes a marriage market and multiple generations. However, where our approach is one of regular, repeated interactions between parents throughout the fertility and child-rearing process, they employ a three-period overlapping generations framework to study these issues. This suits their object of interest, the life-cycle timing of fertility. In contrast, we model spouses' decisions throughout the fertility and child-rearing process, and our ultimate interest is in child outcomes and parents' welfare and their relationship to family law.

A second related paper is Tartari (2015). Tartari addresses the question of whether a child whose parents divorced would have been better off if the parents had remained married. She also models fertility, time and goods investments in a child, and divorce. Relative to our approach, Tartari elaborates the role of conflict within marriage in spouses' child investment decisions and in shaping divorce and child outcomes, as she models an endogenous mode of interaction for married couples and relies on explicit marital harmony or conflict survey instruments from the National Longitudinal Survey of Youth's 1979 cohort (NLSY79). The extent of conflict in the marriage plays a central role in Tartari's inferences regarding whether a child of divorce would be as well off under the counterfactual continued marriage. While Tartari emphasizes conflict data and choices, we focus on time use, observed through detailed time diaries collected in 1997, 2002-2003, and 2007-2008 for both mothers and fathers. Our approach allows us to study parents' explicit time investment in children of all ages, for stable families and for families approaching and experiencing separation and divorce.

In our model, spouses make (simultaneous) choices regarding marriage continuation, fertility, and, where relevant, individual investments in children. A match value of the marriage is drawn from a population distribution and evolves stochastically over time. Fertility choices are influenced by both the expected benefit from the presence of the child and expectations regarding the duration of the marriage, given the state of the marriage quality process. The child progresses as a result of both endogenous parental investment and marital status choices and exogenous productivity factors. Further, child quality is self-productive. In this manner, our child quality production process builds on those of Del Boca, Flinn, and Wiswall (2014) and Cunha and Heckman (2007) by incorporating marriage quality and family structure stability.

Marital dissolution may result from changes in marriage match quality, changes in child presence and quality, and the child reaching maturity. Thus the full history of marriage values and child investments determines current marital status and child investment levels. If the history of child investments and marriage values is poorer for the marginal marriage than it is for the average marriage, then, all else equal, the child welfare gain associated with the continuation of the marginal marriage is smaller than that associated with the continuation of the average marriage. A central objective of our analysis is to study the welfare impacts of variations in family law, which are possible to assess under our assumptions regarding the determination of the utility levels of husbands, wives, and (potential) children. The ability to use the model to infer distinctions between the marginal marriage, which may dissolve in response to changes in prevailing child support, child placement, or divorce standards, and the average marriage is crucial to our ability to recognize distinctions between the divorce rate and family welfare consequences of family policy regimes.

The model is estimated using data from the Panel Study of Income Dynamics (PSID) and its Child Development Supplement (CDS), using the Method of Simulated Moments (MSM). The model we estimate is extremely parsimonious. The primary benefit of this stylized approach is our ability to infer from our short vector of estimated parameters the primary economic processes that drive any particular observed relationship between family outcomes and policies. The natural drawback to parsimony is often a deficiency in model fit. Nevertheless, we find that the model is able to fit the features of the data used in estimation satisfactorily, with a few notable exceptions. The accuracy of fit gives us some confidence in using the estimated model to perform comparative statics exercises and welfare analysis.

The parameter estimates generated by this MSM estimation themselves reveal several new insights regarding the production of children's cognitive ability. Chief among these is the finding that the influence on the growth of child cognitive ability of persisting in a low quality marriage is substantially worse than the influence of realized divorce on the growth of child cognitive ability. In addition, our parameterization of child cognitive development allows us to study the producitivity of parents' time investments as children age. We find that mothers' and fathers' time is highly productive in advancing children's cognitive ability in early childhood, but that the productivity of these time investments declines steeply as children age toward independence. Finally, as in Cunha and Heckman (2007) and others, children's current cognitive ability is estimated to be highly productive in generating future cognitive gains. Further, our estimates indicate that this relationship is relatively stable as the child ages.

Our analysis concludes with a series of counterfactual policy experiments, in which we manipulate dimensions of family law and simulate the responses of divorce, fertility, and children's cognitive ability to the law changes using the model under the parameter values estimated using our PSID-CDS sample and the prevailing family law regime. An important feature of our model is the incorporation of a fertility decision. In the comparative statics exercises we find that family law potentially has an important impact not only on the achievement levels of children from intact and non-intact households, but even more fundamentally on the number of children born and the characteristics of the households having them.

We first simulate a move from the unilateral divorce standards that have been widely adopted by US states over the past decades, and that characterize our baseline model, to a more restrictive (and outdated) bilateral divorce standard, and, for comparison, to an outright divorce ban. Our simulation results indicate that, while these movements toward restricting divorce lead to somewhat higher fertility rates and much lower divorce rates, these come at the cost of lower levels of children's cognitive achievement. (Moreover, others' evidence suggests that limiting divorce may bring dire consequences for spouses locked in dangerous marriages.)

In additional counterfactual policy experiments, we simulate large changes in the child support transferred from fathers to mothers in the divorce state. The fertility, divorce, and child cognitive achievement consequences of these large transfer changes are projected to be comparatively modest. Nevertheless, this family law change is one that both increases fertility and divorce and (very modestly) decreases children's attainment. Overall, our policy simulations indicate that children's cognitive outcomes are not best served either by minimizing divorce or by requiring (implausibly) large resource transfers to the custodial parent in the event of divorce. Despite the extensive evidence that children of divorce, on average, fare worse on several dimensions than children of marriage, a careful analysis of marriage quality heterogeneity, as presented here, demonstrates that the divorce rate among parents is a woefully insufficient policy target where the social objective includes supporting children's cognitive development.

The plan of the paper is as follows. In Section 2, we describe some important patterns in the data which we seek to capture in our model. Section 3 develops the details of the model. Section 4 presents the PSID and CDS data in detail, shows descriptive statistics from our sample, describes our estimation method, discusses the manner in which primitive parameters are identified, and reports the estimates of the primitive parameters and our assessment of model fit. In Section 5, we describe our various counterfactual policy experiments and interpret and decompose the simulated effects of changes to family law. Section 6 concludes.

2 The Empirical Dynamics of Divorce and Child Investment

In this section we document important patterns in the data that shine some light on the dynamics among divorce, the cognitive development of children and parental investment. Although the relationships we discuss here are not necessarily causal, they inform our development of the model and are certainly suggestive of particular causal processes. A key challenge for this paper, and one that will ultimately influence our policy experiments, is decomposing the developmental effect of divorce on children.[3]

The state of divorce itself may present adversities because parents lose the benefit of public consumption, while the constraint on time inputs may tighten both because of the custody allocation and because of the increase in the return to labor supply for mothers, who are assumed to be the custodial parent in the case of divorce. Alternatively, children may suffer also because of the family conflict and parental disagreement that often precipitates divorce. Quantifying this trade-off is key for developing optimal family law.

The first piece of empirical evidence on this topic can be found in Figures 1 and 2. Figure 1 shows child test scores, normalized by age, as a function of the time to divorce.[4] A negative time to divorce indicates that the marriage has already dissolved. As we can see, cognitive performance appears to improve in the years after divorce. This at least indicates that the data are inconsistent with a model in which only the state of divorce harms child development. Figure 2, which shows the age-normed growth in test scores between 1997 and 2002 as a function of time to divorce, tells a similar story. In this case, we see a U-shape pattern in developmental growth. Children who are most proximate in time to their parents' divorce display the lowest level of growth in test scores, while children whose parents are equidistant from this point in either state seem to perform comparably. Once again, this story appears to indicate that the process of divorce itself plays a causally important role in development.

Another interesting pattern appears when we inspect the dynamics of parental time investment. Figures 3 and 4 show the measured time inputs (normalized by age) of Mothers and Fathers, respectively, as a function of time to divorce. We see that mean daily hours do in fact decrease when parents enter the divorce state, which is consistent with the story behind the change in the constraints on parents' consumption and labor supply decisions.

[3] This is the key question posed also by Tartari (2015).

[4] Normalization is important because, among other things, it removes the role played by the co-movement between child age and the likelihood of divorce. We adopt this approach throughout this section.

However, it is also evident that this effect begins to emerge in the years prior to divorce. This provides one potential explanation, in addition to the likely increase in family conflict and decline in the quality of family processes, for the observed patterns in test scores.

In Figure 5 we plot normalized test scores against the age of the child in the year their parents divorce. The relationship is monotonic: children whose parents divorce earlier appear to do better than those whose parents divorce later in their life. This is once again consistent with a model in which exposure to unstable marriages has harmful developmental impacts.

We have chosen language carefully in this section so as to not suggest specious causal arguments. The dynamic patterns discovered here can potentially be explained by systematic covariation with important economic observed or unobserved variables. In the next section we develop the model used in the analysis, which clearly represents the mechanisms that we consider to be causal. In addition, we allow for observable and unobservable heterogeneity to also explain the observed patterns in the data.

3 A Model of Child Investment and Divorce Decisions

There exist two decision-makers in our model, spouses $s = 1, 2$. The model is set in discrete time, with one period in the model corresponding to one year in the data. There are five state variables that are relevant to the intertemporal decision-making and payoffs of the two individuals. These are the pair of wages (w_1, w_2) that each can earn in the market, the state of the marriage, θ, the age of a potential child, a, and the child's cognitive skill level, k.[5] Marriage quality can take values on the grid $\Theta \subset \mathbb{R}$, with $\theta = \emptyset$ if the couple is divorced. As a matter of notation, we say that θ takes values in the space $\Theta \cup \{\emptyset\}$, and we let $\chi_d = \mathbf{1}\{\theta \notin \Theta\}$ be an indicator for divorce, with $\mathbf{1}(x)$ taking the value 1 if x is true. If the couple has not yet had a child, we set $a = -1$ and $k = \emptyset$. We let $x = \{w_1, w_2, \theta, a, k\}$ denote the state vector of the decision problem. We write the utility function of spouse s in each period as

$$u_s(c_s, l_s, k, \theta) = \alpha_{s,1} \ln(c_s) + \alpha_{s,2} \ln(l_s) + \mathbf{1}\{a \geq 0\}(\alpha_{s,3} \ln(k) + \zeta) + (1 - \chi_d)\theta \qquad (1)$$

where c_s represents the market good consumption of spouse s and l_s is the leisure consumption of spouse s. The parameters $\{\alpha_{s,1}, \alpha_{s,2}, \alpha_{s,3}\}$ are the spouse-specific preference weights on consumption, leisure, and child quality, and ζ is a constant welfare cost or benefit of child presence unrelated to their quality.[6] We assume throughout that the price of consumption is fixed at 1. Consumption in the marriage state is assumed to be public,

[5] We only consider cognitive development in this paper. Recent papers that have investigated the non-cognitive development of the child include Cunha et al. (2010) and Del Boca et al. (2022)

[6] The parameter ζ can take positive or negative values. It may be interpreted as a welfare cost or benefit of child presence or, equivalently in this specification, as a scaling factor relating the value of child quality to the value of consumption.

so that when $\chi_d = 0$, $c_s = c = w_1 h_1 + w_2 h_2,\ s = 1, 2$. For example, for a married couple with no children,

$$u_s(c, l_s, \theta) = \alpha_{s,1} \ln(c) + \alpha_{s,2} \ln(l_s) + \theta,\ s = 1, 2.$$

Each spouse receives a wage offer each period w_s, $s = 1, 2$, and the married household with a child is subject to the following resource and time constraints

$$\begin{aligned} w_1 h_1 + w_2 h_2 &= c, \\ l_1 + \tau_1 + h_1 &= 1, \\ l_2 + \tau_2 + h_2 &= 1, \\ h_1, \tau_1 &\geq 0 \\ h_2, \tau_2 &\geq 0 \end{aligned} \tag{2}$$

where τ_s is the time invested in child quality production by spouse s. Note that we have normalized the total time available to either spouse to equal 1. This constraint set is the same for married individuals without a child, but in this case $\tau_1 = \tau_2 = 0$ by definition.

We assume that a child can only be born while the husband and wife remain married, and that divorce is an absorbing state. When a child is born, their initial cognitive ability endowment is given by $k_0 \in \mathbb{R}_+$. The growth process of the child's cognitive ability is described by

$$\begin{aligned} k_{a+1} &= \psi_a(\theta) \tau_1^{\delta_{1,a}} \tau_2^{\delta_{2,a}} k_a^{\delta_{k,a}} \\ \Rightarrow \ln k_{a+1} &= \ln \psi_a(\theta) + \delta_{1,a} \ln(\tau_1) + \delta_{2,a} \ln(\tau_2) + \delta_{k,a} \ln k_a \end{aligned} \tag{3}$$

The presence of marriage quality, θ, in the child quality production function is meant to capture the impact of the home environment on the effectiveness of a given level of parental investments. We assume that divorced and married parents share the same child quality production function and thus, production in divorce is defined by an additional TFP parameter, ψ_d. The Cobb-Douglas shares of inputs $\delta_{s,a}$ and the TFP parameters are permitted to depend on the age, a, of the child. This modeling assumption reflects known developmental differences across stages of the child's developmental cycle. Notice that this specification implies that once a child is born, each parent will spend some time investing in the child, since $\tau_s \to 0 \Rightarrow \ln k_{a+1} \to -\infty$.

If the couple divorces without having a child, there is nothing further that connects the individuals in terms of welfare, since we do not consider the possibility of alimony, only child support transfers. In the case of a divorce without children, the former couple loses the scale economies of sharing the same household but no longer suffers the disutility associated with a poor marriage quality.

If the couple divorces after having a child, their utilities are interconnected throughout the remainder of the child-rearing period, and not only due to the loss of scale economies in consumption. Both divorced parents continue to value the child's cognitive ability. In addition, child support orders serve as transfers that reduce one parent's income while increasing the other's. With some loss of generality, we define spouse 1 to be the husband and the payer of child support in the divorced parenting state. When divorced, spouse 1 is ordered to pay a given percentage of their income to spouse 2, who is granted no less than 50 percent of physical custody.[7] This reduces the father's income and increases the mother's non-labor income.

Child custody in the divorce state is considered to be an additional constraint imposed upon the time allocation choices by family law. We assume that parents have full access to the child in the marriage state, so that when $\chi_d = 0$, τ_1 and τ_2 are unconstrained (subject to the overall time endowment of each spouse, which is 1). When $\chi_d = 1$, assuming that $\overline{\tau}_s$ represents the share of physical custody allocated to parent s in the divorce state, the custody requirement is

$$\tau_1 \leq \overline{\tau}_1 \text{ and } \tau_2 \leq \overline{\tau}_2,$$

with $\overline{\tau}_1 + \overline{\tau}_2 \leq 1$. We further assume that physical custody and visitation allocations are fully anticipated and set exogenously with respect to parental behaviors. [8]

As is apparent from (2), we assume that each individual spouse has no non-labor income when married or when divorced with no children.[9] In divorce, consumption is no longer public. Divorced parents consume only their own incomes, and parent 1 is generally required to pay child support to parent 2. As is the case in many U.S. states, the order amount is determined as a proportion of the father's income, the "tax rate" being denoted as π.[10] Each spouse has a baseline income flow at any moment in time of $w_s h_s$. The actual income under the control of individual s is state-dependent in the following sense: the father pays share π of his income to the mother in the divorced parenting state. Hence, the father's consumption in divorce after time decisions are made is $(1-\pi)w_1 h_1$, and the mother's is $w_2 h_2 + \pi w_1 h_1$.[11] We denote the post-transfer resources of parent s in the

[7] We assume that there is perfect compliance with child support orders on the part of the father. Del Boca and Flinn (1995) develop and estimate a model in which child support orders and compliance decisions are determined endogenously.

[8] See, for example, Fox and Kelly (1995) for details on custody determination.

[9] This assumption is made for computational simplicity and because the data on non-labor income for married couples do not allow for precise assignments of non-labor income sources to each of the spouses. Del Boca et al. (2014) find that most PSID households with children in their sample have little non-labor income.

[10] One would expect some relationship between the mother's share of custody and the level of child support she receives from the father. Rather than imposing a functional relationship between $\overline{\tau}_1$ and π, in the empirical exercise we impose the model custody allocation and base π on predominant child support guidelines and family characteristics. Our model estimation is performed assuming the prevalent child support tax rate, and later policy experiments investigate the effects of reforming to the current maximum among existing state child support guidelines, as well as to a rate far outside of the politically feasible range.

[11] By assuming that there is no transfer ordered after a divorce if the couple is childless, we are essentially assuming away alimony. Alimony is increasingly uncommon in U.S. divorce cases. According to Case et al. (2003), for example, 5 (4.2) percent of 1977 PSID (in 1997) mothers received alimony.

divorced parenting state as $y_s(w_1, w_2, h_1, h_2|\pi)$.

The dynamics of the model are as follows:

1. The model begins at the time of marriage. Spouses are initially childless. If both spouses agree to attempt to have a child, then a child arrives in the next period with probability $\gamma_f > 0$. If the parents attempt to have a child and are unsuccessful, they are free to try again the following period or to reconsider their decision and choose not to have a child in that period.

2. There are N_θ possible values of marriage quality, with $\theta \in \Theta = \{\theta_1, ..., \theta_{N_\theta}\}$ and $\theta_1 < \theta_2 < \ldots < \theta_{N_\theta}$. At the onset of marriage, there is an initial marriage quality draw. Marriage quality evolves stochastically according to a Markov process, i.e. $\theta_{t+1} \sim F_\theta(\cdot \mid \theta_t)$

3. There are N_w possible baseline wage levels for each spouse, with

$$w_s \in \mathcal{W}_s = \{w_{s1}, ..., w_{s,N_w}\},$$
$$0 < w_{s,1} < ... < w_{s,N_w},\ s = 1, 2.$$

   Each spouse begins marriage with a wage of w_s. Similarly to marriage quality, each wage evolves stochastically according to to a Markov Process, i.e. $w_{s,t+1} \sim F_{w,s}(\cdot \mid w_{s,t})$.

4. Child quality in the period is determined by the production process described in (3).

5. The child attains functional independence at age 17, in which case the child quality production process ends. The parents enjoy a terminal value that increases with the current child quality level and continues to depend on the parents' marital status and wages.

Thus, the model is solved over an infinite horizon, with divorce acting as the absorbing state for all agents. This terminal stage can be reached either before or after having a child. If the couple bears a child, then the divorce state becomes terminal once the child has matured at $a = 17$.

The child quality production function described by dynamic elements 4-5 can be related to leading models of child investment. Cunha and Heckman (2007) and Cunha, Heckman, Lochner, and Masterov (2006) argue that a variety of skills that children must develop are subject to "critical periods" early in life, and hence much of intellectual development is accomplished by the time the child reaches school age. Hopkins and Bracht (1975), for example, demonstrate that IQ is stable by the age of 10 or so, suggesting that the critical period for intellectual development occurs by this time. Further, Cunha and Heckman, Cunha et al., and Cunha, Heckman and Schennach (2010) emphasize the importance of both cognitive and non-cognitive skill acquisition to child outcomes, along with the importance of "dynamic complementarity" and "self-productivity" of skill levels in ongoing skill production. Todd and Wolpin (2003, 2007) consider cognitive skill formation, and argue from a different perspective for the importance of both current and lagged inputs to the ongoing production process. They demonstrate the importance of allowing for unobserved endowment effects and the endogeneity of inputs to child skill production.

Like Todd and Wolpin (2003, 2007) and Del Boca et al. (2014, 2016), we restrict attention to cognitive skill. In our empirical implementation, we relate k_t to cognitive test scores from the CDS in a way that permits noisy measurement. The initial conditions that we specify when estimating the model directly address the need to account for unobserved endowment heterogeneity, and the model accounts for endogeneity of investments in determining absolute skill level in a specific manner. Finally, the investment problem that we model begins at birth. Our empirical implementation focuses on progress from birth through a set of tests that are completed for most sample children before the age of ten.

In modeling the behavior of married and divorced parents, an important specification choice is the manner in which spouses interact. One may assume that spouses interact cooperatively or noncooperatively.[12] It is unclear that ex-spouses are able to interact in a manner that results in efficient outcomes that lie on the Pareto frontier. In a model that moves though married and divorced states, if cooperation is ever attained in marriage it is unclear how spouses' mode of interaction might transition from such cooperation in marriage to the potential cooperation failures of divorce, or how the presence of children might influence interactions in the divorce state. One might assume cooperation throughout, but this skirts important implementation issues.[13] We assume noncooperative interaction throughout the model, independent of the state of the marriage or the presence of children. More complex approaches include allowing spouses to choose the current mode of interaction as events progress, following Flinn (2000), or specifying population heterogeneity in spouses' mode of interaction, following Eckstein and Lifshitz (2009) and Del Boca and Flinn (2011). Though the latter approaches are appealing, they would add a great deal of complexity to an already complex model. For the above reasons, and given evidence that divorce requirements (i.e., whether unilateral or bilateral) have impacts on divorce (see, e.g., Friedberg (1998), Gruber (2004)), we have chosen to assume noncooperative interaction throughout. Finally, we assume that spouses' investment strategies constitute a Markov Perfect Equilibrium.[14]

We now turn to the detailed consideration of the dynamic programming problem faced by household members as it varies according to the marital and child-rearing stages of the household. In what follows, let $V_s(x)$ be the value function of spouse s in state x. We

[12]For examples of the cooperative and non-cooperative approaches, respectively, see Browning and Chiappori (1998), Lundberg and Pollak (1994), and Del Boca and Flinn (2011).

[13]It is sometimes argued that cooperative agreements are sustainable in households because all actions chosen by household members are potentially observable by other members of the household. This constant monitoring makes it possible to implement agreements that are not best responses. When the parents of a child do not cohabit, this monitoring is not possible making the implementation of cooperative agreements quite problematic.

[14]See, for example, Pakes and McGuire (2000).

allow the vector of decision variables to be given by $\gamma = [\gamma_1, \gamma_2]$, which belongs to a space $\Gamma(x)$ that depends on the current state, x. We can write the generic dynamic programming problem as:

$$V_s(x) = \max_{\gamma_s \in \Gamma(x)} \{u_s(c_s, l_s, x) + \beta \mathbb{E}[V(\hat{x}) \mid \gamma_1, \gamma_2, x]\} \tag{4}$$

This problem can be solved under provision of the strategy profile of the other spouse $\gamma_{3-s}(x)$ and the transition equations that govern the Markov transition kernel. A Markov Perfect Equilibrium in this model is a set of strategies $\{\gamma_1, \gamma_2\}$ and value functions $\{V_1, V_2\}$ such that the pair (V_s, γ_s) solves the dynamic program above, subject to the strategies of the other spouse, γ_{3-s}. This condition holds simultaneously for $s = 1, 2$. We state this problem in more detail by describing how the transitions and constraint set Γ vary with the state variable x.

3.1 Divorced Parents

Given the absence of a remarriage market, divorce is an absorbing state. Since the value of the future is independent of current labor supply, an ex-spouse s who has a child of quality k at the termination of the investment process need only solve the equivalent of a static labor supply problem. The relevant state variables in this case are the wage of individual s, w_s, and the final attributes of the child, k. The value function is defined recursively by:

$$V_s(w_1, w_2, \emptyset, 17, k) = \max_{h_s} \left\{u_s(w_s h_s, 1 - h_s, x) + \beta \mathbb{E}_{\hat{w} \mid w} V_s(\hat{w}_1, \hat{w}_2, \emptyset, 17, k)\right\} \tag{5}$$

Here, we can embed the constraints of the problem and the transition rules directly within the dynamic program. Notice that w_s and k are the only elements of the state vector that are relevant to period utility. Furthermore, since the child is mature, k is fixed and time-invariant.

In the case of divorce with an ongoing child quality improvement process, each parent's time allocation problem now includes the decision of how much to invest in the child, with the evident interdependence of actions between the parents and the implications for the future value of the problem facing each parent. We define an equilibrium in parental time investments and labor supplies, which is determined by the state of the marriage, child quality, and parental wages. In addition, since child development is a finite-stage process, the age a of the child is a state variable for the parental investment problem. Since we are using a Cournot-Nash equilibrium framework in a dynamic setting, each parent s takes the strategies of the other parent s' as given. Following the logic of strategic equilibrium, each parent then solves the dynamic problem:

$$V_s(w_1, w_2, \emptyset, a, k) = \max_{l_s, h_s, \tau_s} \left\{u(c_s, 1 - h_s - \tau_s, x) + \beta \mathbb{E}[V_s(\hat{w}_1, \hat{w}_2, \emptyset, a+1, \hat{k}) \mid \tau_1, \tau_2, w_1, w_2, k]\right\}$$

subject to

$$\begin{aligned}
\hat{k} &= \psi_{a,d}\tau_1^{\delta_{1,a}}\tau_2^{\delta_{2,a}}k^{\delta_{k,a}} \\
c_s &= y_s(w_1, w_2, h_1, h_2|\pi) \\
l_s + h_s + \tau_s &= 1 \\
h_s, l_s &\geq 0 \\
\tau_s &\leq \overline{\tau}_s
\end{aligned}$$

Here, V_s represents the value to spouse s, which depends on the wages of each parent, (w_1, w_2), the child's current attributes, k_a, and the age, a, of the child. The expectation next period is taken with respect to the Markov transition kernels described above. Solution of this dynamic program produces a pair of strategies (τ_s, h_s) that are functions of the state, x. The concept of Markov Perfect Equilibrium requires that the strategies of the other spouse, $(\tau_{-s,a}, h_{-s,a})$ taking this pair as given, solve the equivalent problem.

3.2 Married Parents

The experiences they will have if they enter the divorce state can meaningfully affect the investment decisions of forward-looking married parents. The derivation of the married parents' equilibrium is similar to that of the divorced parents' equilibrium, with one major difference being the search for an equilibrium in divorce decisions as well as in investments and values. As before, we begin with the value of a terminated child investment process at k_{17} for spouse s. Spouses solve the same static labor supply problem as in the divorce state with terminated investment, except that their earned income contributes to the public household consumption c. In addition to their labor supply decision, spouses make a "divorce request" decision, d_s.

It is necessary for us to specify the manner in which divorce decisions are made. Under our assumption of noncooperative behavior, these decisions are not, in general, efficient. The nature of the decisions depends critically on legal statutes. There are two polar cases upon which we focus. The first is *unilateral* divorce, in which it is enough for one of the parents to ask for a divorce for the couple to enter the divorce state. The second is termed *bilateral* divorce, and in this case both parents must agree to divorce for divorce to occur. In the model, the dissolution standard is a function $\mathbf{d}$ that maps the pair (d_1, d_2) to the set $\{0, 1\}$, where 1 represents a divorce and 0 the opposite. We can formalize these two standards as follows:

$$\mathbf{d}(d_1, d_2) = \mathbf{1}\{d_1 + d_2 > 0\} \tag{Unilateral}$$

$$\mathbf{d}(d_1, d_2) = d_1 d_2 \tag{Bilateral}$$

The mass movement of states out of bilateral to unilateral divorce standards took place primarily in the late 1960s and early 1970s, well before the era in which our sample spouses married. During the relevant years for our sample, the large majority of states, though not all, maintained unilateral divorce standards. Given the time frame of the sample we use, we solve and estimate the model under the assumption of unilateral divorce. However, the bilateral divorce standard is clearly a politically relevant possibility for family law, and one that decreases the ease with which spouses can access divorce. We consider the bilateral case in some of our counterfactual exercises, out of interest both in the family welfare impacts of this particular politically relevant divorce standard and, more broadly, in the family welfare and child attainment impacts of making divorce more difficult to obtain.

In terms of the timing of the divorce decision, we allow parents to make their divorce decisions, (d_1, d_2), *after* the realization of each period's marriage shocks. We begin by considering the divorce decisions of parents of children for whom the parental investment process has ended. In this case the only choices of parents are labor supply and whether to divorce. We write the choice of d_s in the following dynamic program within the maximization operator next period. This yields

$$V_s(w_1, w_2, \theta, 17, k) = \max_{h_s} \left\{ u_s(c, 1 - h_s, x) + \beta \mathbb{E} \max_{d_s} \left[V(\hat{x}) \mid d_1, d_2, x \right] \right\},$$

subject to the constraints:

$$\begin{aligned} c &= w_1 h_1 + w_2 h_2 \\ \hat{x} &= \{\hat{w_1}, \hat{w_2}, \emptyset, 17, k\} && \text{if } \mathbf{d}(d_1, d_1) = 1 \\ \hat{x} &= \{\hat{w_1}, \hat{w_2}, \theta, 17, k\} && \text{if } \mathbf{d}(d_1, d_2) = 0. \end{aligned}$$

The labor supply decisions of the parents in the period are made in accordance with a simultaneous move game.

When the child investment process is still active, parents make time investment as well as labor supply decisions. Much like the case in divorce, this introduces a dynamic element to the non-cooperative game. At age a, each spouse takes the current and future strategies of the other as given. These are restricted to be a function of the relevant state variables. We can therefore define these equilibrium conditions recursively. Let $V_s(w_1, w_2, \theta, a, k)$ indicate the value to spouse s in a marriage of quality θ, with wages (w_1, w_2) and an a-aged child of quality k_a. This stage of the problem can be stated as follows:

$$V_s(w_1, w_2, \theta, a, k) = \max_{\tau_s, l_s, h_s} u_s(c, l_s, x) + \beta \mathbb{E} \max_{d_s} \left[V_2(\hat{x}) \mid d_1, d_2, \tau_1, \tau_2, x \right]$$

subject to

$$
\begin{aligned}
c &= w_1 h_1 + w_2 h_2 \\
\hat{x} &= \{\hat{w}_1, \hat{w}_2, \emptyset, a+1, \hat{k}\} && \text{if } \mathbf{d}(d_1, d_1) = 1 \\
\hat{x} &= \{\hat{w}_1, \hat{w}_2, \hat{\theta}, a+1, \hat{k}\} && \text{if } \mathbf{d}(d_1, d_2) = 0 \\
\hat{k} &= \psi_a(\theta) \tau_1^{\delta_{1,a}} \tau_2^{\delta_{2,a}} k^{\delta_{k,a}} \\
1 &= \tau_s + h_s + l_s
\end{aligned}
$$

It is apparent that each state variable has a potential influence on the divorce decisions of the couple. This means that investment and labor supply decisions themselves may be influenced by the degree to which they influence the probability of divorce in future periods. Thus, the general specification of this model permits a highly complex set of causal interactions. In the appendix, we will show that the solution to this Markov Perfect Equilibrium given our modelling assumptions admits a far simpler solution, which facilitates analysis and estimation.

3.3 Childless Couples and the Fertility Decision

Since divorce is an absorbing state and the fertility process is only active in the married state, a divorced childless ex-spouse s solves the same static labor supply problem as does a divorced spouse with a mature child. This stage of the dynamic program satisfies the recursion:

$$V_s(w_1, w_2, \emptyset, -1, \emptyset) = \max_{h_s} \{u_s(w_s h_s, 1 - h_s, x) + \beta \mathbb{E}[V_s(\hat{w}_1, \hat{w}_2, \emptyset, -1, \emptyset) \mid x]\} \tag{6}$$

Childless married couples, on the other hand, must jointly choose to continue in the marriage and attempt to conceive a child, to continue in the marriage and not attempt to conceive a child, and whether to divorce. This requires us to adopt a timing convention, which is the following. In period t, parents decide whether or not to attempt to have a child. If they do make such an attempt, a child arrives next period with probability γ_f. We allow parents to make their divorce decision in $t+1$ *after* the realization of the fertility shock. Thus, we can write the dynamic program in the following way:

$$V_s(w_1, w_2, \theta, -1, \emptyset) = \max_{h_s, f_s} u_s(c, 1 - h_s, x) + \beta \mathbb{E} \max_{d_s} [V_s(\hat{x}) \mid d_1, d_2, f_1, f_2, x]$$

subject to

$$
\begin{aligned}
c &= w_1 h_1 + w_2 h_2 \\
\hat{x} &= \{\hat{w}_1, \hat{w}_2, \tilde{\theta}, \hat{a}, \hat{k}\} \\
\tilde{\theta} &= \hat{\theta} && \text{if } \mathbf{d}(d_1, d_2) = 0 \\
\tilde{\theta} &= \emptyset && \text{if } \mathbf{d}(d_1, d_1) = 1 \\
\{\hat{a}, \hat{k}\} &= \{0, k_0\} && \text{w.p. } \gamma_f f_1 f_2 \\
\{\hat{a}, \hat{k}\} &= \{-1, \emptyset\} && \text{w.p. } 1 - \gamma_f f_1 f_2.
\end{aligned}
$$

Each spouse s solves a maximization problem over labor supply, h_s, that takes the decision of the other as given. As discussed previously, we require that this pair of choices (h_1, h_2) satisfies the constraints of strategic equilibrium. The continuation value from a successful fertility attempt integrates over possible realizations of child quality, k_0, future wages, and marriage quality, subject to the divorce policies of each spouse. A similar integration is performed over possible state realizations in the case of an unsuccessful fertility attempt, which occurs with probability $1 - \gamma_f$, or with probability 1 if fertility is not attempted. Finally, this attempt itself is the result of a *bilateral* decision: it is chosen if requested by both partners, according to their strategies (f_1, f_2).

The model described in this section includes fertility decisions through the arrival of the first child only. Subsequent fertility is unmodeled, though estimation of the model will include those families that eventually have more than one child. We have chosen subsequent fertility as a dimension on which to limit the scope of the modeling exercise, as the modeling of ongoing fertility equilibria, and investment allocation among many siblings, complicates the specification more than it adds to available insights on marital status and child investment dynamics.[15] However, the resources invested and the decision to divorce are surely influenced by the number of children. In Section 2, as in other research, we observed that marital stability shows a strong positive association with fertility. A primary channel through which unmodeled subsequent fertility is likely to affect our results is the larger family size, and therefore likely smaller investment per child, in intact households. The omission of investment in second and later children is likely to mute any effect of divorced parenting on the firstborn children's attainment that is implied by our model.

3.4 Solving and Characterizing the Equilibrium of the Model

As discussed above, in its general form this model allows for a complex set of strategic and dynamic interactions. This may, in principle, make the equilibrium decision rules difficult

[15] Del Boca et al. (2014) do not model fertility decisions, but they do investigate the investment decisions of parents in intact households when they have two children. As they make clear, it is necessary to make a number of strong assumptions regarding the production technology in order to estimate the model. Their model did not include fertility or divorce decisions, which makes the sets of assumptions required even more stringent and the computational problem unwieldy.

to characterize. However, under the specifications that we use here, we will see that this model permits considerable simplification. In this section we will discuss the nature of these decision rules and present some intuition for the results. We relegate the technical details of the solution to the appendix.

First, it can be shown that for couples with children, the value functions V_s for $s = 1, 2$ are additively separable in log-child quality, $\log(k)$. Specifically, we have the following:

$$V_s(w_1, w_2, \theta, k) = (1-\beta)^{-1}\alpha_{s3}[\log(k) + \zeta] + \mathcal{V}_s(w_1, w_2, \theta, a) \tag{7}$$

$$\alpha_{V,s,a} = \alpha_{s3} + \beta\delta_{3,a}\alpha_{V,s,a+1} \tag{8}$$

$$\alpha_{V,s,17} = (1-\beta)^{-1}\alpha_{s3} \tag{9}$$

An immediate consequence of this result is that, in Markov Perfect Equilibrium, labor supply, divorce and investment strategies are not a function of child quality. In addition, the closed form solution of this component of the value function greatly simplifies solution of the model, since k is a continuously distributed state variable. This allows for a much richer set of empirical implications than would otherwise be possible. In Appendix 7 we show how this result can be derived in addition to the form of the component value functions $\mathcal{V}_s$. Intuitively, the derivation proceeds as follows. First, when the child reaches maturity, $\log(k)$ is static, and hence enters as an additive constant, as is shown in equation (7). While the child is developing, there is a period return to $\log(k_a)$ represented by α_{3s}. In addition, $\log(k_a)$ enters additively in the production of $\log(k_{a+1})$, which recursively preserves additive separability. These two components represent the value to $\log(k_a)$ in the current period, which is shown in (8).

One important consequence of this simplification is that the return to time investments is itself log-linear, which greatly simplifies the investment problem. At each stage of the dynamic program, we can take first order conditions with respect to h_s and τ_s (when applicable) to produce a low-dimensional set of equations that characterize equilibrium choices. First, when spouses are divorced without a child or with a mature child, labor supply solves a static choice problem:

$$h_s = \frac{\alpha_{1s}}{\alpha_{1s} + \alpha_{2s}} \tag{10}$$

In the case of married couples without a child or with a mature child there are also no dynamic considerations, hence labor supply is also a static choice, but must constitute a strategic equilibrium. First order conditions produce the pair of equations:

$$h_1 = \max\left\{1 - \phi_1 - \phi_1\left(\frac{w_2 h_2}{w_1}\right), 0\right\}$$

$$h_2 = \max\left\{1 - \phi_2 - \phi_2\left(\frac{w_1 h_1}{w_2}\right), 0\right\}$$

$$\phi_s = \frac{\alpha_{2s}}{\alpha_{1s} + \alpha_{2s}}$$

When parents have an actively developing child, each must choose τ_s in addition to h_s, factoring in the future returns in child quality to investments. As mentioned above, simplifying the value functions makes the first order conditions to this problem more tractable. When parents are divorced, the problem is no longer symmetric, since the father must pay a fraction π of his income in child support.[16] The father's first order conditions yield the solution:

$$h_1 = \frac{\alpha_{11}}{\alpha_{11} + \alpha_{12}}(1 - \tau_1) \tag{11}$$

$$\tau_1 = \min\left\{\overline{\tau}_1, \frac{\delta_1 \beta \alpha_{1,V,t+1}}{\alpha_{11} + \alpha_{12} + \delta_1 \beta \alpha_{1,V,t+1}}\right\}, \tag{12}$$

while, given this choice, first order conditions for the mother produce the solution:

$$h_2 = \max\left\{\frac{\alpha_{21}}{\alpha_{21} + \alpha_{22}}(1 - \tau_2) - \frac{\alpha_{22}}{\alpha_{21} + \alpha_{22}} \frac{\pi h_1 w_1}{w_2}, 0\right\} \tag{13}$$

$$\tau_2 = \min\left\{\frac{\delta_2 \beta \alpha_{2,V,t+1}}{\alpha_{22} + \delta_2 \beta \alpha_{2,V,t+1}}(1 - h_2), \overline{\tau}_2\right\} \tag{14}$$

Finally, when parents are married, the solution to the problem bears strong resemblance to the solution without active development:

$$h_1 = \max\left\{\frac{\alpha_{11}}{\alpha_{11} + \alpha_{12} + \delta_{1,a} \beta \alpha_{V,1,a+1}} - \frac{\alpha_{12} + \delta_{1,a} \beta \alpha_{V,1,a+1}}{\alpha_{11} + \alpha_{12} + \delta_{1,a} \beta \alpha_{V,1,a+1}} \frac{h_2 w_2}{w_1}, 0\right\} \tag{15}$$

$$\tau_1 = \frac{\delta_{1,a} \beta \alpha_{V,1,a+1}}{\alpha_{12} + \delta_{1,a} \beta \alpha_{V,1,a+1}}(1 - h_1) \tag{16}$$

$$h_2 = \max\left\{\frac{\alpha_{21}}{\alpha_{21} + \alpha_{22} + \delta_{2,a} \beta \alpha_{V,2,a+1}} - \frac{\alpha_{22} + \delta_{2,a} \beta \alpha_{V,2,a+1}}{\alpha_{21} + \alpha_{22} + \delta_{2,a} \beta \alpha_{V,2,a+1}} \frac{h_1 w_1}{w_2}, 0\right\} \tag{17}$$

$$\tau_2 = \frac{\delta_{2,a} \beta \alpha_{V,2,a+1}}{\alpha_{22} + \delta_{2,a} \beta \alpha_{V,2,a+1}}(1 - h_2) \tag{18}$$

The solution for labor supply is functionally identical to the childless case, with coefficients adjusted for the extra marginal value of a unit of time. In addition, time investment is allocated according to its proportional value with leisure.[17]

[16] Our specification of the child support process is somewhat at odds with what happens in actual divorce cases involving children. In most cases, the order is set at the time of the final divorce decree as a function of parental incomes. These orders are only changed periodically to reflect changes in the income levels of the parents and usually occur after bringing a formal legal action. In addition, compliance with actual child support orders is far from perfect, as shown in Del Boca and Flinn (1995). If the child support order was viewed as a fixed amount that was periodically revised rather than as a tax and if compliance with an order was viewed as a choice variable, the model would clearly be much more complex.

[17] These results are similar to those in Del Boca et al. (2014) for the simpler case of unitary utility of the household, no divorce, and no fertility decision.

4 Estimation and Results

4.1 Data

To answer the empirical and policy questions outlined in Section 1, we use data from the Panel Study of Income Dynamics (PSID) and its Child Development Supplement (CDS). The PSID is a dynastic, longitudinal survey conducted annually from 1968 to 1997, and biennially since 1997. It collects information on a range of economic and demographic indicators. The CDS consists of three waves, collected in 1997, 2002 and 2007. Any child in a PSID family between the ages of 0 and 12 at the time of the 1997 survey was considered eligible. These surveys contain a broad array of developmental scores in cognitive and socioemotional outcomes, as well as information on the home environment of the child. One crucial feature of the survey for our purposes is the availability of time use data, which is collected from the participants using detailed time diaries. We provide further details below.

4.2 Description of Variables and Sample Selection

From the PSID's primary survey we collect data on mothers' and fathers' labor supply, labor income, total family income, and some demographic variables. The CDS is comprised of several questionnaires. We use two in particular: the child interview and the primary caregiver (PCG) interview. From the child interview, we draw our measure of cognitive ability based on the Letter-Word (LW) module of the Woodcock-Johnson Aptitude test. The cognitive ability measure that we incorporate in the model estimation is a record of the number of items on the LW module that the child answers correctly, out of a possible 58.[18]

In addition, the CDS asks each participant child to fill out a time diary. This portion of the survey requires participants to record a detailed, minute by minute timeline of their activities for two days of the week: one random weekday and one random weekend day. Activities are then coded by PSID time diary specialists at a fine level of detail. When necessary, children are assisted in completion of the time diary by the PCG. These diaries provide detail on the daily lives of children that is unparalleled among economic data resources.

From the coded time use diaries, we construct a measure of maternal time investment by taking a weighted sum of the total hours of time use in which the mother is recorded as actively participating in each diary activity. Similarly, we construct a measure of paternal time investment by taking a weighted sum of the total hours of time use in which the father is recorded as actively participating in each diary activity. A key advantage of the PSID CDS data is this balanced evidence on the time with children of both mothers and fathers.

[18]There are actually 57 items on the test, with the possible number of correct answers ranging from 0 to 57.

Other valuable survey resources containing detailed economic data provide extensive detail on mothers' time investment in children, but they omit or make asymmetric or limited record of the time invested in child development by fathers. Further, the presence of fathers' time in the data is, in some cases, contingent on the parents' continued marriage. In the PSID CDS, we are able to measure time investment from married, divorced, and single mothers and fathers, and this feature of the PSID CDS data is crucial to our analysis.

Since the focus of this project is on initially married parents, and their fertility and divorce decisions, along with their distinct child investment choices, we restrict our sample to parents who are married in the first wave of the PSID CDS in 1997. This leaves us with a sample of 959 mothers, and 1,405 children. Table 1 reports descriptive statistics for the estimation sample. Wives in the PSID CDS initially married sample are, on average, roughly 2.5 years younger than husbands at the time of marriage, at 26.66 and 29.11, respectively. Wives and husbands have similar mean schooling levels at marriage, of 12.7 and 12.6 years, respectively. Approximately half of the sample marriages result in children during the observation window, who are represented in the CDS. The mean time from marriage to the arrival of children, conditional on arrival, is 2.5 years. The average number of years from marriage to divorce is 10.9 years. Note, however, this average contains right censored observations of marriages that are ongoing at the final survey date in 2007. We observe a cumulative sample divorce rate of 15.44 percent by 2007. Women earn approximately 12 USD per hour in 1997 and 14 USD per hour in 2002, while men earn 17 USD per hour in 1997 and 20 USD per hour in 2002, with all financial variables reported in terms of 2007 USD. Moreover, while men work nearly 50 percent more daily hours than women do in this realtively young estimation sample focused on fertility and childrearing decisions, women spend considerably more than 50 percent more hours per day in child care than men spend, and these findings hold in both 1997 and 2002.

4.3 Parameterization of the Model

In this section we discuss the parameterization of the model, which is necessary before we consider identification.

4.3.1 Preferences

The preference parameters describing the model are the Cobb-Douglas coefficients (α_1, α_2) for the spouses, the flow utility ζ associated with having a child, the marriage quality value θ, and the discount factor β. Each vector of Cobb-Douglas parameters α_s contain three values. The first value is associated with the consumption of spouse s, the second is associated with the leisure of spouse s, and the final value $\alpha_{s,3}$ appears only when a child is present. In this case, the utility flow from having a child of ability k is given by $\alpha_{s,3}\ln(k) + \zeta$. The utility flow from having a child is the same for both parents and is received independently of the state of the marriage, meaning that it remains even in the event of a divorce.

The marriage quality process is modeled as follows. The spouses share the same value of marriage quality at each point in time. At every point in time marriage quality is assumed to take one of five possible values, $\theta^{(k)} = -1 + 0.5(k-1),\ k = 1, 2, ..., 5$. For purposes of defining the probability distribution of θ, we will set $\theta^{(0)} = -\infty$. We allow the marriage quality distribution to depend on the educational level of the spouses, which reflects patterns observed in the likelihood and timing of divorce. We define $E_s = 1$ if individual s has more than 12 years of schooling, with $E_s = 0$ if the individual has only completed high school or less. At the beginning of the marriage, period 1, there is an initial draw of marriage quality θ_1, with the probability distribution of θ_1 given by

$$P(\theta_1 = \theta^{(k)}|E_1, E_2) = \frac{\Phi\left(\frac{\theta^{(k)} - \mu_\theta^0 - \mu_\theta^1 E_1 - \mu_\theta^2 E_2}{\sigma_\theta}\right) - \Phi\left(\frac{\theta^{(k-1)} - \mu_\theta^0 - \mu_\theta^1 E_1 - \mu_\theta^2 E_2}{\sigma_\theta}\right)}{1 - \Phi\left(\frac{\theta^{(5)} - \mu_\theta^0 - \mu_\theta^1 E_1 - \mu_\theta^2 E_2}{\sigma_\theta}\right)},$$

$$k = 1, 2, ..., 5.$$

As long as the couple stays married, the marriage quality process continues. The process is first-order Markovian, with the probability of $\theta_{t+1} = \theta^{(k)}$ conditional on $\theta_t = \theta^{(j)}$ given by

$$P(\theta_{t+1} = \theta^{(k)}|\theta_t = \theta^{(j)}) = \frac{\Phi\left(\frac{\theta(k) - \rho_\theta \theta^{(j)}}{\sigma_\theta}\right) - \Phi\left(\frac{\theta(k-1) - \rho_\theta \theta^{(j)}}{\sigma_\theta}\right)}{1 - \Phi\left(\frac{\theta^{(5)} - \rho_\theta \theta^{(j)}}{\sigma_\theta}\right)}$$

$$k = 1, 2, ..., 5.$$

Thus there are five parameters that characterize the marriage quality process, the three μ parameters that appear in the distribution for θ_1, the persistence parameter ρ_θ that characterizes the transition matrix, and the scale parameter σ_θ that appears in both.

Finally, the discount factor β is clearly an important preference parameter in a model of investment. We assume that both marriage partners share the same value of β. Given the challenges we face in estimating the model we did not try to estimate the discount factor, instead we make the common assumption that $\beta = 0.95$ since the decision period length is one year.

4.3.2 The Wage Processes

Due to the fact that we are not able to obtain closed form solutions for the model as in Del Boca et al. (2014), it is necessary for us to discretize the wage distributions of the husbands and wives. For computational simplicity we have also chosen to ignore the receipt of non-labor income by the husband and wife by assuming that neither individual has non-labor income. The only exception is when the couple are divorced with a child, in which case the mother will receive non-labor income in the form of child support from the father.

We allow husbands and wives to have 8 distinct values of wages, $\mathcal{W}_{s,j},\ j = 1, 2, ..., 8$. The discrete wages are determined by using the gender-specific observed wage distributions for the years 1997 and 2002, given by $R_s(w)$. Then we define

$$\mathcal{W}_{s,j} = R_s^{-1}(j/9),\ j = 1, 2, ..., 8,$$

where R_s^{-1} is the inverse of R_s. As we did when defining the probability distribution of θ, let $\mathcal{W}_{s,0} = -\infty$. Then we assume that the probability that an initial wage draw w_s is equal to $\mathcal{W}_{s,j}$ is given by

$$P(w_{s,1} = \mathcal{W}_{s,j}) = \frac{\Phi\left(\frac{\mathcal{W}_{s,j}-\mu_{w,s}}{\sigma_{w,s}}\right) - \Phi\left(\frac{\mathcal{W}_{s,j-1}-\mu_{w,s}}{\sigma_{w,s}}\right)}{1 - \Phi\left(\frac{\mathcal{W}_{8,j}-\mu_{w,s}}{\sigma_{w,s}}\right)}$$
$$j = 1, 2, ..., 8.$$

We have not conditioned on schooling in the distribution of the initial wage so as to reduce the number of parameters to be estimated. However, we do allow the growth path of wages to be a function of schooling since it is well-established that the more highly-educated have higher growth rates in wages and earnings. We assume that wages follow a random walk, with the probability of individual s moving up from $\mathcal{W}_{s,j}$ to $\mathcal{W}_{s,j+1}$ given by $\Delta^s_{w,1+E_s,+}$ for all $j < 8$. The probability of moving down one wage level from $\mathcal{W}_{s,j}$ to $\mathcal{W}_{s,j-1}$ is denoted $\Delta^s_{w,1+E_s,-}$ for all $j > 1$. If the individual is at the highest wage level, $\mathcal{W}_{s,8}$, they only face the risk of moving down, and those at the lowest wage level, $\mathcal{W}_{s,1}$, only face the risk of moving up.

4.3.3 The Child's Cognitive Ability Production Function

We recall that the (log) child ability production function, equation (3), is given by

$$\ln k_{a+1} = \ln \psi_a(\theta) + \delta_{1,a} \ln(\tau_1) + \delta_{2,a} \ln(\tau_2) + \delta_{k,a} \ln(k_a)$$

at age a. As in Del Boca et al. (2014), we restrict the Cobb-Douglas parameters to be monotonic functions of age, so that

$$\delta_{i,a} = \exp(\gamma_{\delta,0,i} + \gamma_{\delta,a,i} a),\ i = 1, 2, k.$$

The TFP parameter in the production technology depends on the marriage quality given marriage, the age of the child, and whether the parents are divorced. We can succinctly write the logarithm of TFP as

$$\ln \psi_a(\theta) = \psi_d \chi_d + \psi_0(1 - \chi_d) + \gamma_{\psi,\theta}\theta(1 - \chi_d) + \gamma_{\psi,a} a.$$

Then the production technology is characterized by 10 parameters in all, 4 of which parameterize TFP as a function of the child's age and marriage quality. The other 6 parameters characterize the productivity of each of the three inputs in the child ability production technology, the time inputs of the parents, and the child's previous cognitive ability.

4.4 Measuring Child Cognitive Ability and Divorced Fathers' Time

We infer the child's latent cognitive ability, k, from the Woodcock-Johnson Aptitude test's Letter Word score (LW) described above. In doing so, we follow the steps of Del Boca et al. (2014); their section on inferring latent child cognitive ability from the observed test score provides a comprehensive description of our method. Put briefly, for our present purposes, we anchor the child's latent cognitive skills at age a to the probability of responding correctly to an item on the LW at age a. Hence we assume that the LW score, LW, given child latent ability k is distributed as

$$LW \sim \text{Binomial}\left(\frac{k}{1+k}, 58\right).$$

Here the probability of a correct response on each LW item increases with latent ability, and, at the same time, our approach allows each fielding of the aptitude test to represent child cognitive ability imperfectly.

A second element of the empirical implementation of the Section 3 model of fertility, child investment, and divorce that requires that we introduce some form of measurement error in mapping the model to the data is, naturally, divorced fathers' time with children. In the large majority of surveys that include parent-child connections, divorced fathers are either entirely absent, or, in the most favorable cases, partially observed. Beginning our sample window from the date of marriage, and therefore excluding never-married parents, has the obvious drawback of failing to represent the child investment decisions of a non-negligible, and not randomly excluded, minority of parents. However, we have chosen to track couples from marriage in order to have some chance of observing fathers through both marriage and divorce. The time use diaries of the PSID CDS children do represent the time investments in children of a large number of divorced fathers. However, children's time use is recorded on a randomly determined weekday and a weekend day. On average, in the divorce state, divorced fathers appear, in these measurements, to spend less time in the presence of their children than do divorced mothers or married mothers or fathers. We infer from this pattern and survey method that divorced fathers who spend some time with their children are stochastically captured by the randomly chosen weekday and weekend day diary.

In response to this partial time diary success in capturing divorced fathers' time investments, we build some structure into our empirical exercise that is intended to describe the data's imperfect ability to represent divorced fathers' time investments. We assume that the time diary captures the father with the child with probability $\overline{\tau}_1$, where $\overline{\tau}_1$ is the father's physical custody share, and otherwise the time diary records zero time with the father.

4.5 Identification

Since we will be using a moment-based estimator and not a maximum likelihood estimator, we cannot provide a rigorous proof of point identification of all of the parameters in this relatively complex dynamic model. One of the advantages of using a Method of Simulated Moments estimator is that the estimation procedure is built from the Data Generating Process (DGP) associated with the model. The parameterization of the model that was discussed in Section 4.3 was in fact chosen in part so as to match certain dynamic and cross-sectional patterns observed in the data. We will try to convey something about the (rough) mapping between the parameters and the data features that enable us to identify them with a reasonable degree of accuracy (according to the bootstrap standard errors).

We begin with the preference parameters. In the case of a married household with no children, conditional on the wage draws of the husband and wife, (w_1, w_2), the leisure decisions of the household are determined by the intersection of their reaction functions, which are only a function of $(\alpha_{1,1}, \alpha_{1,2}, \alpha_{2,1}, \alpha_{2,2})$. Thus the labor supply decisions of these households, conditional on wage draws, are informative regarding these preference parameters. Unlike in Del Boca et al. (2014), we have assumed homogeneity of preference parameters in the population of married households so that potential selection issues for sub-groups, such as those married with no children, do not arise.

The parameters of the marriage quality process admittedly are more challenging to identify since marriage quality θ has no effect on labor supply decisions in households with no children. Marriage quality does have an impact on the likelihood of having children, however, in combination with the wages of the spouses and the value of a (independently-distributed) fertility value shock. Given the form of preferences and the production technology, marriage quality also impacts the growth in child quality through its impact on the TFP term in the production technology. It does not have a direct impact on the child investment decisions of the parents under our functional form assumptions, meaning that more subtle impacts of marriage quality on investment decisions are not present. The production of child quality in the divorce state is independent of the previous path of child quality draws, meaning that changes in child quality around the time of divorce, as TFP moves from the value associated with marriage (at perhaps a low marriage quality) to that associated with divorce, are key to estimating the TFP parameters associated with (unobserved) marriage quality. Once again, TFP in the divorce state will not influence the investment decisions of divorced parents.

We have assumed that fertility value shocks, ζ, are household specific and are drawn from a Normal distribution with mean μ_ζ and standard deviation σ_ζ. These parameters are quite difficult to identify, since they mainly serve to induce a married couple to have a child.[19] However, in conjunction with the wage processes, which are partially observed, and the marriage quality process, which is largely identified through divorce outcomes and changes in the cognitive test scores that we utilize, identification is possible. For example, a household in which both spouses have high wages may be more likely to have a child since the marginal utility of consumption is declining in income, meaning that the probability of getting a fertility value shock that induces a birth is higher than in a couple in which the marginal utility of consumption is lower. Differences in birth outcomes between households differing in wages and marriage quality values is key to identifying the two parameters characterizing the distribution of ζ.

Turning now to the identification of the parameters characterizing the wage processes for the husbands and wives, the problem is much more straightforward. With loss of generality, we have assumed that the household members have no non-labor income. This implies that if the individuals were in single-person households with no non-labor income both would supply time to the labor market, meaning that their wages would be observed in each period of time available to us in the PSID sample. In a household setting in which consumption is a fully public good, this of course need not be the case, particularly when a child is present. Nonetheless, for a substantial proportion of the spouses in the sample we observe wages in every period in which they could possibly be observed. Within intact marriages, marriage quality has no impact on the labor supply decisions of the couple. Coupled with our assumption that all individuals in the population of married couples share the same preferences, this implies that the selection process on wages that operates through equilibrium outcomes in which one spouse is not working is transparent. This makes the identification of the 12 parameters characterizing the dynamic wage processes of the husbands and wives relatively straightforward.[20]

We conclude our discussion of identification by considering the parameters characterizing the production process of child cognitive ability. Here we can draw on the analysis in Del Boca et al. (2014) in which a slightly more elaborated production function of the same functional form was estimated using MSM. Given our parameterization of the production function, we have

$$\begin{aligned} \ln k_{a+1} = \psi_d \chi_d + \psi_0 (1 - \chi_d) + \gamma_{\psi,\theta} \theta (1 - \chi_d) + \gamma_{\psi,a} a \\ + \exp\left(\gamma_{\delta,0,1} + \gamma_{\delta,a,1} a\right) \ln(\tau_{a,1}) \\ + \exp\left(\gamma_{\delta,0,2} + \gamma_{\delta,a,2} a\right) \ln(\tau_{a,2}) \\ + \exp\left(\gamma_{\delta,0,k} + \gamma_{\delta,a,k} a\right) \ln(k_a) \end{aligned} \tag{19}$$

[19]The fertility process is considered to be deterministic in the sense that once a couple decides to have a child there will be a birth with probability 1. The model itself is well-defined even when the probability of a birth, γ_f, is less than 1. However, if we allow the birth outcome to be probabilistic function of the fertility decision we would have to estimate at least one additional parameter, γ_f, which worsens the identification problem with regards to the distribution of ζ.

[20]Del Boca et al. (2014) were able to estimate fewer parameters of the wage processes of the parents since they were not forced to discretize wages so as to solve their simpler model. We are forced to estimate more parameters than they did in order to adequately capture the dynamics in the discretized wage processes.

The most ideal situation for identification of the parameters of this function is when the TFP parameter is stochastic, with

$$\ln \tilde{\psi}_a(\theta) = \ln \psi_a(\theta) + \nu_a,$$

where ν is i.i.d. with mean 0 and standard deviation σ_ν. Recall that under our modeling assumptions TFP has no impact on the investment decisions of the parents, thus

$$E(\nu_a|\ln \tau_{a,1}) = 0$$
$$E(\nu_a|\ln \tau_{a,2}) = 0$$

Now $\ln k_a$ will be a function of ν_{a-1} so that if there is persistence in the TFP shocks then ν_a will not be mean independent of $\ln k_a$, but if the TFP shocks are i.i.d., then

$$E(\nu_a|\ln k_a) = 0.$$

In this case, if we had access to annual observations on the $\ln k$ process, observed the marriage quality values at each point in time, and if there was sufficient variation in divorce states and child ages, the parameters of the child cognitive ability production process would be identified and could be consistently estimated using a nonlinear least squares estimator.

Unfortunately we do not have access to these ideal data and face at least two challenges. Firstly, we do not have access to annual observations on any of the observables in (19). Not only is the PSID a biennial survey after 1997, complicating the estimation of the annual wage and labor supply processes, but all of the variables appearing in Equation (19) are collected only within the Child Development Supplement (CDS), which was fielded in 1997 and 2002. Thus in actuality we only have one growth measure for each child, which compares their ability measure in 2002 with their ability measure in 1997.

This problem is one of the main reasons we use a simulation-based estimator, which allows us to deal with the missing data problem created by infrequent sampling. As in Del Boca et al. (2014), we simulate a path of wages for the husband and wife beginning from an initial draw at the beginning of the marriage and then simulating a sequence of wages over the period modeled. We do the same for the marriage quality shocks, where the initial draw is taken in the year in which the marriage takes place and then evolves according to the random walk process for marriage quality that we specified in Section 4.3.1. Given the simulated exogenous stochastic processes for wages and marriage quality over the period 1997 to 2002, we compute the child quality in 2002 given the child quality in 1997 and the sequence of annual decisions and outcomes over the five-year period. Thus we employ the DGP associated with the model along with current trial values of the parameters to continuously update the sequence of inputs and choices over the period 1997 to 2002 to form the mapping from $k_a(1997)$ to $k_{a+5}(2002)$.

The other issue we face is the fact that k is unobserved. Instead, we have only one indicator of $k_a(1997)$ and one indicator of $k_{a+5}(2002)$, which are the cognitive test scores in those two years in which the CDS was fielded. We have discussed the manner in which we map cognitive test scores into k in Section 4.4. This mapping does not rely on estimates of any of the primitive parameters of the model.

4.6 Simulation

The model is simulated using the empirical distribution of initial marriage years and education levels of each spouse. Using 10 simulations per marriage, wage and marriage quality levels are drawn according to the initial distributions described in Sections 4.3.1 and 4.3.2. The simulation models the exact sample selection of the CDS: moments are calculated using only the subset of simulated household panels with a child between the age of 0 and 12 in 1997.

4.7 Moments Used in Estimation

We estimate the model described in section 3, using the sample of 959 PSID families and 1,405 PSID CDS children described in subsection 4.1, through an application of the Method of Simulated Moments (MSM). Tables 2 through 5 each begin by reporting sample values of a series of fertility and divorce (Table 2), wage and labor (Table 3), and child development (Table 4) moments that our model simulation will target.

Interpreting the content of these three tables requires us to establish some notation. Let Y_D indicate the year of divorce and Y_B the year the child is born. S_Y is the test score for the child in year Y and $\tilde{S}_Y$ is the same score normalized by age. Similarly, $\Delta S = S_{02} - S_{97}$ is the growth in test scores over this time period, while $\widetilde{\Delta S}$ is the same change, normalized by age. Finally $\tilde{\tau}_{s,Y}$ is the time investment of spouse s in year Y, normalized by age. $Y_D - Y$ is a variable that indicates the number of years until divorce, while D_Y is a binary variable that is equal to 1 if the couple has divorced by year Y. $\mathcal{W}_s$ is the wage grid used in the model for spouse s and $\mathcal{W}_{s,j}$ is the jth point on this grid. Finally, Q_q is the operator that computes the qth quantile of a variable.

The Table 2 child development moments in the data that our simulation will seek to match include mean test scores among children ages five and under, children between five and 10, and children between 10 and 14; mean time investments from mothers and fathers in the cognitive development of children in each of these age groups; quantiles of the score distributions for children in these younger and older age groups; and expectations of the products of early and late scores with parents' time inputs early and late. These moments serve primarily to identify the child cognitive development production parameters, including the marriage quality-dependent TFP parameter $\psi_{a,\theta}$, and its relationship to divorce state TFP parameter ψ_d, as well as the various $\delta_{s,a}$ parent-by-age time productivity parameters.

The Table 3 fertility and divorce moments targeted by the model simulation include the share of families in which divorce occurs by the 5th year, and the share who divorce by the tenth year; the share experiencing a birth by the second year of marriage, and also the fifth; several interactions between the marriage duration until the first birth and the marriage duration until divorce; moments describing the calendar timing of birth and divorce through the 2010 end of the program; and divorce rates by 2010 that condition on the wife's education or the husband's education. These moments contribute to the identification of the fertility process parameter, the marriage quality process parameters, and, more indirectly, the host of parameters governing wages and children's cognitive development.

Finally, our wage and labor moments, listed in Table 4, include mean hours of husbands and of wives in both 1997 and 2002; moments describing the distribution of working wives and husbands across each group's wage grid; and expected wages among wives and husbands that condition on their education levels. These moments directly identify the wage process for both spouses, and the preference parameters governing the relative utility from consumption, leisure, and child cognitive ability, and contribute indirectly to the identification of parameters governing the production of child cognitive ability.

4.8 Estimation Results

Estimates from the MSM procedure are shown in Table 5, along with standard errors computed using a bootstrap estimator. Given the detailed child cognitive ability production function, wage processes, and evolution of marriage quality in the model, these parameter estimates may be most coherently discussed using a series of figures. Figure 6 compares the values of ψ_d with the expression $\psi_0 + \gamma_{\psi,\theta}\theta$ for different values of θ. Importantly, we see that the gradient on θ is positive: low quality marriages are less productive of cognitive ability. Furthermore, we see that $\psi_{a,d}$ is greater than $\psi_a(\theta)$ for low realizations of θ. At the best-fitting vector of child cognitive ability production parameters emerging from our MSM estimates, the TFP for the lower half of the set of possible marriage qualities lies below the TFP for the divorce state. This feature of the estimates has implications for growth rates in child test scores in the years preceding divorce, in which the quality of marriage will be lower on average. This pattern is found in the data, in Figure 2, and will be discussed later with respect to model fit.

Figure 7 depicts the estimates of the Cobb-Douglas shares of paternal time, maternal time, and current cognitive ability $(\delta_{1,a}, \delta_{2,a}, \delta_{k,a})$ in the production of future cognitive ability. We find that the influence of both fathers' and mothers' time is substantial but decreases with age, which is consistent with findings in Del Boca et al. (2014). In contrast however, we find that fathers' time is slightly more influential at all ages. The estimated differences in the contributions of mothers' and fathers' time to child cognitive development are small and, particularly at young ages, apt to be within the margin of statistical error. Additionally, we find that the share of current ability in the production of future cognitive ability is large, beginning just above 1.0, and it is notably persistent as the child ages. The share of current skill in production decreases with age, but the magnitude of the decrease is quite small and unlikely to be empirically relevant.

Turning to the parents' objectives, Table 5A reports the best-fitting vector of preference parameters for husbands and wives. Our estimated preference weights on consumption, $\alpha_{1,1}$ and $\alpha_{2,1}$, are 0.7428 and 0.6823 for husbands and wives, respectively. Leisure preference weights, $\alpha_{1,2}$ and $\alpha_{2,2}$, are 0.4156 and 0.3363. Hence we estimate that husbands place more preference weight on consumption and, by a wider margin, on leisure than wives do. Wives' preferences instead tilt more heavily toward child quality, with estimated child quality coefficients in the parents' objectives, $\alpha_{1,3}$ and $\alpha_{2,3}$, of 0.0352 and 0.0575. Thus, at this vector of best-fitting parameters, our model explains the absolute and relative levels of mothers' and fathers' time investments in children by attributing a slightly higher child quality productivity rate to time with the father than to time with the mother, but also a substantially greater preference weight on child quality for mothers than for fathers.

It is worth noting that our estimated preference weights on child quality are markedly lower than those estimated by Del Boca et al. (2014), despite our use of nearly identical data and scale of measurement. The two models differ in several ways. Where Del Boca et al. (2014) use a unitary objective with pooled consumption, husband's leisure, wife's leisure, and child quality as arguments, we maintain distinct objectives for husbands and wives. Most importantly, the introduction of divorce in our model expands the set of measures identifying each parent's preference weight on child quality. While in Del Boca et al. the preference weight on children is principally determined by the parents' levels of time investment, under our model the preference weight on children can also be determined in part by the relative rates of divorce of couples with and without children and the sensitivity of divorce to the marriage quality process. Hence the smaller magnitude of the preference weights on child quality that we estimate need not imply a less consequential role for child quality in spouses' decision-making in our model than in Del Boca et al. (2014).

The wage processes of the mothers and fathers at the best-fitting vector of wage process parameters match characteristics of the empirical wage distributions in several expected ways. The Table 5C wage process estimates suggest the existence of a high wage type and a low wage type for both mothers and fathers, and that these are predictably related to education levels. Fathers move up the wage ladder with probability 0.77 for the high type and with probability 0.69 for the low type. High type mothers ascend with probability 0.69 and low type mothers ascend with probability 0.67. Thus, fathers ascend the wage ladder more quickly than mothers. The estimates also suggest that there is greater heterogeneity in wage outcomes for fathers than for mothers. These parameters imply stochastic growth in the wages of both spouses over time, which is necessary in order to match the observed growth in individual wages with age in the data. Thus, the Markov process reflects not just time-invariant wage risk but also expected wage growth over the life-cycle.

In order to evaluate model fit, the simulated moments from the model, evaluated at the estimated vector of parameter values in Table 5, can be compared with the empirical moments in Tables 2, 3, and 4. First, in Table 2, we compare the child development moments measured for our 1405 sample children with the child development outcomes arising from our model simulation. The child cognitive ability production process is the heart of our story, and, as such, we are most interested in capturing the development of children's cognitive abilities over the course of early and middle childhood, and its connection to mothers' and fathers' time inputs, accurately. In Table 2, we find that the broad patterns in the data, in terms of scores earlier and later in childhood, mothers' and fathers' time with their children earlier and later in childhood, and the products of scores and time both early and late, are reproduced by the simulation. The match between sample moments and simulated moments is particularly close during middle childhood. For example, the mean LW score is 30.95 in the data and 30.15 in the simulation in 1997, and 48.13 in the data and 46.50 in the simulation in 2002. Daily hours with the mother for 6- to 10-year-olds are 3.28 and 2.75 in the 1997 data and model, respectively, and 1.90 and 2.05 in the 2002 data and model. Analogous hours with the father are 1.84 and 1.98 in the 1997 data and model, and 1.47 and 1.69 in the 2002 data and model. The most distant match between data and model in Table 2 appears to be in the extent of the dispersion of the test scores of older children; for example, while the 25th and 75th percentiles of the 1997 test score distribution in the data are 42 and 50, the simulated 25th and 75th percentiles of the 1997 scores are 34 and 58.

In Table 3, the model simulated divorce rates, and divorce rates by parents' education levels, are lower than the observed divorce rates in 1997 but match realized divorce rates, overall and by education subgroup, by 2010. Fertility rates also begin somewhat more slowly in the model than in the data. However, by five years of marriage the model share of marriages that have experienced first births is 39.45 percent and the data share is 44.72 percent. By ten years from the date of marriage, the model overshoots the data fertility target, with 49.00 percent of marriages in the data and 60.52 percent of marriages in the model having experienced births. Next, the product of divorce and fertility at 3, 5, 7, and 10 years of marriage shows a related slow start in the model relative to the data, but a close fit by 10 years of marriage, with 2.93 percent of the sample and 2.75 percent of model simulated families experiencing divorced parenting by 10 year from the date of marriage. Divorce realizations continue throughout the sample window, with the final share divorced by 2010 reaching 15.44 percent in the data and 17.02 percent in the simulation.

Finally, the wage offer process is well fit for both working wives and working husbands by the several parameters governing the model wage offer process, and this is reflected in the similarity between the data and model moments in Table 3. The distribution of workers across six wage bins for mothers and six wage bins for fathers is closely matched point-by-point in 1997, and wages evolve similarly enough in the model and data that their mean levels conditional on education group are again closely matched by 2002. The largest gap between model and data in Table 3 is in the rate of growth in hours worked from 1997 to 2002 for wives, and for husbands, with the growth rates in hours somewhat lower in the data than in the model simulation. Hours levels are, however, quite similar from data to model, with mean daily hours near six in the 1997 and 2002 model and data for husbands and near four in 1997 and 2002 model and data for wives.

5 Policy Experiments

We now use point estimates of the model to simulate welfare and child outcomes under counterfactual policy arrangements. In this section we study two elements of family law that are particularly relevant to couples' choices of fertility, child investment, and the continuation of a marriage, and that may therefore shape children's cognitive development and family members' welfare. We accomplish this by simulating outcomes for each couple in the sample under alternative policy arrangements, and we assume that these alternative policies remain in effect from the first year of marriage until their children reach maturity. We study two policy changes of interest. The first is moving from a unilateral to a bilateral divorce standard, which, for most states, constitutes a reversal of recent divorce law reforms. The second policy change we study is an increase in the rate of child support taxation on fathers in the divorced parenting state, π. We increase π from the 15 percent child support tax rate assumed in our estimation, which was drawn from the modal rate generated by state child support guidelines for the time period and sample we study (for single child families), first to 30 percent, which is the upper bound of plausible tax rates based on state child support guidelines and relevant family structures, and second to a comparatively extreme, and likely politically infeasible, 45 percent.

In order to assess the impacts of these policies, it will be useful to exploit the log-additivity of the production function in investments. In particular, note that we can write final child skills as:

$$\log(k_{17}) = \sum_{a=0}^{16} \overline{\delta}_a \left[\log(\psi_a(\theta)) + \delta_{1,a}\log(\tau_{1,a}) + \delta_{2,a}\log(\tau_{2,a})\right] + \overline{\delta}_0 \log(k_0),$$

where $\overline{\delta}_a = \prod_{s=a+1}^{16} \delta_{k,s}$. Let P_a denote the measure over realized state variables (θ, w_1, w_2) for a child at age a, which is defined over the space $\Theta \cup \{\emptyset\} \times \mathcal{W}_1 \times \mathcal{W}_2$. By noting that τ_1 and τ_2 are determined only by the wages and whether or not the couple is divorced, we can calculate expected outcomes as:

$$\mathbb{E}\left[\tilde{k}_{17}\right] = \sum_{a=0}^{16} \overline{\delta}_a \left[\sum_{\theta \in \Theta \cup \{\emptyset\}} P_a[\theta]\psi_a(\theta) + \sum_{w_1, w_2} P[w_1, w_2]\delta_{1,a}\tilde{\tau}(w_1, w_2) + \delta_{2,a}\tilde{\tau}_2(w_1, w_2)\right]$$

By forcing this measure to sum to 1, we get:

$$\mathbb{E}\left[\tilde{k}_{17}\right] = \text{const} + \sum_{a=0}^{16} \overline{\delta}_a \left[\sum_{\theta \in \Theta} P_a[\theta](\psi_0 + \gamma_\psi \theta - \psi_d) + \delta_{1,a}\Big(P_a[d=0]\mathbb{E}_a[\tilde{\tau}_1 \mid d=0] + (1 - P_a[d=0])\mathbb{E}_a[\tilde{\tau}_1 \mid d=1]\Big) + \delta_{2,a}\Big(P_a[d=0]\mathbb{E}_a[\tilde{\tau}_2 \mid d=0] + (1 - P_a[d=0])\mathbb{E}_a[\tilde{\tau}_2 \mid d=1]\Big)\right] \quad (20)$$

This expression suggests that we can interpret the impact of policy changes on child skills through three channels: through marriage quality, through maternal time investment, and through paternal time investment. The first term describes the effect of a policy change through total factor productivity. Although marriage qualities evolve exogenously in the model, the distribution of marriage qualities and marital status among those couples with children in equilibrium is molded through each couple's fertility and divorce policies. If the family law reform changes the realized fertility at each marriage quality level, or if it changes the stability of marriage at each marriage quality level among those couples raising children, then it will change the contribution of marriage quality and marital status to the cognitive development of children via total factor productivity.

The second and third terms in the above equation represent the effect of the family law reform on time investment, which can operate in two ways. First, holding the probability of divorce at each age fixed, a policy may impact time investment policies by changing the incentives of each member of the couple to work. An increase in child support, for example, may both increase the mother's choice of hours with the child and decrease the father's choice of hours with the child for those couples in the divorced parenting state. Second, since there are differences in time investment between married and divorced parents, changes in the composition of parents across the married and divorced states will also affect time investment in their children. If the family law reform decreases the divorce rate, as we might expect both a move to the bilateral marriage dissolution standard and a sharp increase in child support tax rates to do, then it will change the time investments of the marginally surviving couples from the time investment levels arising in the divorce state to those arising in marriage.

We will find that each of these three components, marriage quality effects, mother's time investment effects, and father's time investment effects, make relevant contributions to the policy responses we simulate in the following sections.

Finally, we can collect terms to calculate a decomposition of mean effects on child skill outcomes. Rewriting (20) as:

$$\mathbb{E}\left[\tilde{k}_{17}\right] = \underbrace{\sum_{a=0}^{16} \overline{\delta}_a \mathbb{E}_a[\tilde{\psi}_a(\theta)]}_{=\Psi} + \underbrace{\sum_{a=0}^{16} \overline{\delta}_a \delta_{1,a} \mathbb{E}_a[\tilde{\tau}_1]}_{=\mathcal{T}_1} + \underbrace{\sum_{a=0}^{16} \overline{\delta}_a \delta_{2,a} \mathbb{E}_a[\tilde{\tau}_2]}_{=\mathcal{T}_2}$$

Thus, when reporting the change in average child skills that is induced by a policy, this effect can be decomposed into three components. If x is a variable obtained under the baseline policy, we let $\hat{x}$ indicate its counterpart under the new policy:

$$\hat{\mathbb{E}}\left[\tilde{k}_{17}\right] - \mathbb{E}\left[\tilde{k}_{17}\right] = \hat{\Psi} - \Psi + \hat{\mathcal{T}}_1 - \mathcal{T}_1 + \hat{\mathcal{T}}_2 - \mathcal{T}_2 \tag{21}$$

We use this decomposition to analyze the effects of the two policy changes discussed below.

5.1 Bilateral Divorce

We begin with the simulated effects of a counterfactual move to a bilateral divorce standard, in which both spouses must agree to a divorce for the divorce to be granted. Table 6 shows the counterfactual rate of fertility and divorce among couples in the sample under a bilateral divorce standard, as well as the change in average test scores (among those children born under each policy) relative to the baseline case. For comparison, we also simulate outcomes in the case in which divorce is effectively outlawed. We call this the "no divorce" case.[21] As is expected, moving to a bilateral divorce standard substantially reduces the rate of divorce, from a baseline divorce rate of 11.90 percent by 2010 under the unilateral divorce standard to 6.55 percent by 2010 under the bilateral divorce standard. At the same time, the adoption of a bilateral divorce standard leads to a slight increase in the rate of fertility among married couples, from 54.63 percent of marriages experiencing births by 2010 under the unilateral standard to 57.33 percent experiencing births by 2010 under the bilateral standard.

We draw attention to the simulated effect on test scores: reducing the rate of divorce by forcing each party to agree on the divorce decision reduces cognitive outcomes for the children who are born under the new, bilateral divorce standard, in comparison to the average cognitive outcomes of children born to our sample married couples under the unilateral divorce standard. Specifically, the move from unilateral to bilateral divorce standards decreases the mean test score among children born to our full sample of initially-married couples by 0.1007 points. The test score effects of a complete divorce ban are similar, with the movement from unilateral divorce to a full divorce ban leading to a 0.1054 point decrease in average test scores among children born under each divorce regime.

Our decomposition of policy effects into marriage quality, father's time investment, and mother's time investment mechanisms, reported in Table 8, sheds light on the sources of these changes. In the first row of decomposition results, we report the effect of moving from a unilateral to a bilateral divorce regime, decomposed into these three components.

[21] It is worth noting that some of the marriages we observe in our sample would not have occurred under these two increasingly extreme divorce standards. As our model begins at the start of marriage, our simulations based on the estimated model cannot account for the effect of a bilateral divorce standard or marriage ban on the decision to enter marriage.

We expect the decline in divorces to affect child cognitive development through a decrease in the match quality of ongoing marriages, which reduces TFP in the production of child cognitive ability, and through an increase in time parents devote to child development, as a result of the shift from divorce into marriage for marginal couples whose divorce outcomes respond to the reform. Based on our decomposition, the policy appears to significantly lower the average TFP of couples, while only marginally increasing the mean quantity of time inputs. Our accounting in the previous section, and Equation (20) specifically, show that this significant decrease in TFP can only be brought about if the policy lowers the conditional mean quality of marriages. This is achieved by its effect on parental divorce decisions: the probability of divorce in the worst states of marriage is significantly reduced. We indeed find, in our model simulation, that the frequency of divorce at low marriage quality levels decreases substantially with the move from the unilateral to the bilateral divorce standard. As the child ages, we see, in the bilateral case, an increase in the relative frequency of low-quality marriages. The results of this policy present a clear implication: that policies such as bilateral divorce, that inhibit poor marriages from dissolving, will have harmful developmental effects on the children of these marriages.

It is crucial to note that this analysis of the effect of the bilateral, as opposed to the unilateral, divorce standard on children's cognitive development and spouses' welfare focuses on mean child ability and spouses' mean welfare outcomes. Stevenson and Wolfers (2006), for example, demonstrate substantial reductions in suicide, spousal homicide, and domestic violence with the state-by-state movement, in recent decades, from bilateral to unilateral divorce regimes. Our analysis of mean child cognitive ability and mean spouse welfare measures overlooks meaningful distributional effects of these divorce requirements, including the distress experienced by a spouse trapped in an abusive marriage by a restrictive legal regime. From this perspective, what our novel analysis of the influence of marital dissolution standards on fertility, child investment, and divorce adds is evidence that bilateral divorce standards and divorce bans, in addition to harming spouses who seek to leave dangerous marriages, may also damage the cognitive development of children whose parents are locked by the legal regime into low quality marriages.

5.2 Child Support

Following the logic of the previous section, we may pursue other policy initiatives that increase the ease with which couples can harmoniously divorce. If mothers prefer to stay in low quality marriages because of their low outside option when living alone, then this may act as a disadvantage to the child. It is reasonable, then, to ask if increasing financial support to mothers after divorce may enable low quality marriages to dissolve and improve child outcomes.

To evaluate this, we consider two large increases in the rate of child support, π, paid by the father in our model. In particular, we consider taxation rates of 30 percent and 45 percent, which are much higher than the 15 percent used in our baseline analysis. Figure

7 shows the aggregate results of these policy changes. Contrary to the intuition derived from our previous case study, although these changes do increase the divorce rate very slightly, we find that mean child outcomes are (modestly) negatively affected by the large child support increases implemented in this experiment.

Once again, we use the decomposition of equation (21) to look for answers to this puzzle. In Table 8, we report the decomposition of the effect of the change from a 15 percent child support tax rate to a 30 percent child support tax rate into contributions from marriage quality changes, fathers' time investment changes, and mothers' time investment changes. First, note that the changes in average TFP are once again driving a large portion of these results. In particular, we see once again that the policy leads to an increase in average TFP. However, in stark contrast to the previous experiment, we now see a large decrease in the mother's time investment. This loss in the mother's investment outweighs the gain in TFP, producing a net loss in child skills. Interestingly, we see an increase in paternal time investment, which is driven by a reduction in labor supply. This suggests increased dissolution of marriages in which the father was working more, which will occur when their wages are comparatively higher than mothers'.

6 Conclusion

We have developed and estimated a model that allows for strategic behavior between parents in making fertility, child quality investment, and divorce decisions. An important component of the behavioral model is the family law environment, which has a large impact on the rewards attached to the marital states and, in turn, the returns to investment in child quality. We use data from the PSID and the PSID-CDS to estimate model parameters using a Method of Simulated Moments estimation procedure. We find that the parameter estimates are roughly in accord with our priors, and that the correspondence between simulated and sample moments is generally close (though in some subsets of the moments measuring fertility and divorce dynamics the correspondence is merely adequate).

The most important contribution of our work is to the understanding of the dynamic relationship between divorce decisions and the evolution of fertility and child quality, and the dependence of this process on family law parameters. While there is a well-established empirical relationship between children's outcomes and the characteristics of the households in which they live, we have attempted to disentangle the simultaneous relationships among divorce, fertility, and child development using a behavioral model of these decisions. To our knowledge, this is one of the first studies to link the family law environment to the fertility decisions of intact families, and, in some instances, we find the link to be substantial.[22] While our estimated model is based on a number of restrictive and, ultimately, untestable assumptions, our view is that this type of framework is the only way to begin to understand the complex dynamics present within married households.

[22]The other that we are aware of is Aizer and McLanahan (2006).

Our model estimates are useful in their own right, as they allow us to test predictions including the relative child cognitive development productivity of family settings featuring strong marriages, ex post weak marriages, and divorce. Our estimates indicate that the child cognitive ability total factor productivity of low match quality marriages lies below that of divorced families. This result aligns with descriptive evidence demonstrating slow growth of child cognitive ability as parents approach divorce, followed by a recovery of ability growth as the family moves past divorce. In addition, our parameter estimates are able to show us the levels and age dependence of the child cognitive development productivity of their mother's time inputs, their father's time inputs, and their own current cognitive ability. We find that productivity of mothers' and fathers' time inputs to child development is initially high but declines steeply as the child ages toward independence. The child's own cognitive ability is estimated to have a particularly high level of productivity in generating future cognitive ability gains, and this productivity is substantially more persistent as the child ages than are the various parental inputs.

We have conducted investigations of how substantial changes in the parameters characterizing the family law environment - those reflecting the ease with which marriages may be dissolved and the child support transfers between parents in the divorce state - impact fertility, child outcomes, and the distribution of parental welfare. In line with the evidence from the parameter estimates above regarding the low productivity of a weak marriage in producing child cognitive ability gains, counterfactual policy simulations in which we make divorce more difficult to obtain by implementing a bilateral divorce standard, or we ban divorce outright, indicate a meaningful decline in child cognitive attainment under legal regimes that restrict access to divorce. Our estimated model, in various ways, describes a damaging effect on children of remaining in a low match quality marriage that is approaching or barred from divorce.

In addition, we investigate the effects of two substantial increases in the child support paid by fathers to mothers in the event of divorce on children's cognitive achievements. An increase from a 15 percent child support tax rate to a 30 or 45 percent child support tax rate both increases the share of marriages that experience a birth by 2.7 percentage points and increases the divorce rate very slightly. Despite enhancing the resources available to mothers who engage in divorced parenting, this change in child support transfers decreases the average cognitive achievement of children.

In combination, the evidence generated by our manipulation of divorce standards and divorce-state transfers does generate one overarching insight regarding the regulation of divorced parenting: while prior descriptive evidence indicates that children of divorce fare worse on several dimensions that children of marriage, it is not the case that designing family law to minimize divorced parenting unambiguously benefits children. More difficult divorce standards, such as bilateral divorce requirements and divorce bans, may increase births and substantially decrease divorce, but they also lead to a meaningful decrease in the average cognitive attainment of the realized population of children. On the other hand, (much) higher transfers to the custodial parent in divorce are projected to increase both fertility and divorce, but they also lead to small declines in children's cognitive attainment. Our model allows the marginal marriage affected by family law to differ meaningfully from the average marriage. In addition, and importantly, our approach allows stable and unstable marriages to differ in their consequences for children's cognitive development. This approach of formalizing marriage heterogeneity in terms of both marriage stability and child ability production allows us to separate the social goals of raising children in stable, high-investment households and protecting children from the developmental damage that appears to arise from prolonging low match quality marriages.

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Table 1: Descriptive Statistics for the PSID CDS Initially Married Sample

Data	Mean
Mother's Age at Marriage	26.6611
Father's Age at Marriage	29.1169
Mother's Highest Grade Completed	12.7888
Father's Highest Grade Completed	12.6480
Years to First Birth	2.5128
Rate of Marriages with Birth	0.4966
Years to Divorce	10.9234
Rate of Marriages Divorce	0.1544
Mother's Mean Wage - 1997	11.9912
Mother's Mean Wage - 2002	14.3163
Father's Mean Wage - 1997	16.5323
Father's Mean Wage - 2002	20.1664
Mother's Mean Daily Labor Hours - 1997	4.1931
Mother's Mean Daily Labor Hours - 2002	4.3279
Father's Mean Daily Labor Hours - 1997	6.1046
Father's Mean Daily Labor Hours - 2002	6.1884
Mother's Mean Daily Time Input - 1997	3.9895
Mother's Mean Daily Time Input - 2002	2.0671
Father's Mean Daily Time Input - 1997	2.2888
Father's Mean Daily Time Input - 2002	1.4734

Notes: These means are drawn from our sample of PSID CDS ever-married couples observed in 1997, 2002, and 2007.

Table 2: Child Development Moments - Data v. Model

Moment	Data	Model
$\mathbb{E}[S_{97} \mid A_{97} \leq 5]$	8.1376	6.9222
$\mathbb{E}[S_{02} \mid A_{97} \leq 5]$	34.7037	26.4890
$\mathbb{E}[\tau_{1,97} \mid A_{97} \leq 5]$	2.9065	2.5301
$\mathbb{E}[\tau_{1,02} \mid A_{97} \leq 5]$	1.6037	1.9851
$\mathbb{E}[\tau_{2,97} \mid A_{97} \leq 5]$	5.3137	3.9655
$\mathbb{E}[\tau_{2,02} \mid A_{97} \leq 5]$	2.5016	2.8475
$\mathbb{E}[S_{97} \mid 5 < A_{97} \leq 10]$	30.9488	30.1297
$\mathbb{E}[S_{02} \mid 5 < A_{97} \leq 10]$	48.1303	46.4981
$\mathbb{E}[\tau_{1,97} \mid 5 < A_{97} \leq 10]$	1.8351	1.9797
$\mathbb{E}[\tau_{1,02} \mid 5 < A_{97} \leq 10]$	1.4695	1.6942
$\mathbb{E}[\tau_{2,97} \mid 5 < A_{97} \leq 10]$	3.2794	2.7488
$\mathbb{E}[\tau_{2,02} \mid 5 < A_{97} \leq 10]$	1.9019	2.0486
$\mathbb{E}[S_{97} \mid 10 < A_{97} \leq 14]$	45.4591	43.3111
$\mathbb{E}[S_{02} \mid 10 < A_{97} \leq 14]$	51.0052	49.9326
$\mathbb{E}[\tau_{1,97} \mid 10 < A_{97} \leq 14]$	1.8740	1.7602
$\mathbb{E}[\tau_{1,02} \mid 10 < A_{97} \leq 14]$	1.1924	1.5992
$\mathbb{E}[\tau_{2,97} \mid 10 < A_{97} \leq 14]$	2.6255	2.1896
$\mathbb{E}[\tau_{2,02} \mid 10 < A_{97} \leq 14]$	1.4454	1.7087
$Q_{25}[S_{97} \mid A_{97} \leq 5]$	4.0000	2.0000
$Q_{75}[S_{97} \mid A_{97} \leq 5]$	12.0000	10.0000
$Q_{25}[S_{97} \mid 5 < A_{97} \leq 10]$	21.0000	11.0000
$Q_{75}[S_{97} \mid 5 < A_{97} \leq 10]$	41.0000	49.0000
$Q_{25}[S_{97} \mid 10 < A_{97} \leq 14]$	42.0000	34.0000
$Q_{75}[S_{97} \mid 10 < A_{97} \leq 14]$	50.0000	58.0000
$\mathbb{E}[\tilde{S}_{97} \cdot \tau_{1,97}]$	0.0741	0.0966
$\mathbb{E}[\tilde{S}_{02} \cdot \tau_{1,02}]$	0.0906	0.1457
$\mathbb{E}[\tilde{S}_{97} \cdot \tau_{2,97}]$	0.1857	0.1247
$\mathbb{E}[\tilde{S}_{02} \cdot \tau_{2,02}]$	0.1383	0.1824

Notes: These columns compare sample moments to moments calculated from simulations performed using the MSM parameter estimates.

Table 3: Fertility and Divorce Moments - Data v. Model

Moment	Data	Model
$\mathbb{E}[\mathbf{1}\{Y_D \leq 5\}]$	0.0355	0.0046
$\mathbb{E}[\mathbf{1}\{Y_D \leq 10\}]$	0.0855	0.0297
$\mathbb{E}[\mathbf{1}\{Y_B \leq 2\}]$	0.3106	0.1911
$\mathbb{E}[\mathbf{1}\{Y_B \leq 5\}]$	0.4472	0.3945
$\mathbb{E}[\mathbf{1}\{Y_B \leq 10\}]$	0.4900	0.6052
$\mathbb{E}[\mathbf{1}\{Y_D \leq 3\} \cdot \mathbf{1}\{Y_B \leq 3\}]$	0.0027	0.0006
$\mathbb{E}[\mathbf{1}\{Y_D \leq 5\} \cdot \mathbf{1}\{Y_B \leq 5\}]$	0.0107	0.0043
$\mathbb{E}[\mathbf{1}\{Y_D \leq 7\} \cdot \mathbf{1}\{Y_B \leq 7\}]$	0.0177	0.0102
$\mathbb{E}[\mathbf{1}\{Y_D \leq 10\} \cdot \mathbf{1}\{Y_B \leq 10\}]$	0.0293	0.0275
$\mathbb{E}[\mathbf{1}\{Y_D \leq 1997\}]$	0.0841	0.0522
$\mathbb{E}[\mathbf{1}\{Y_D \leq 2010\}]$	0.1544	0.1702
$\mathbb{E}[\mathbf{1}\{Y_B \leq 1997\}]$	0.2854	0.4539
$\mathbb{E}[\mathbf{1}\{Y_D \leq 1997\} \cdot \mathbf{1}\{Y_B \leq 1997\}]$	0.0193	0.0261
$\mathbb{E}[\mathbf{1}\{Y_D \leq 2010\} \mid \text{ED}_f > 12]$	0.1388	0.1643
$\mathbb{E}[\mathbf{1}\{Y_D \leq 2010\} \mid \text{ED}_f \leq 12]$	0.1669	0.1750
$\mathbb{E}[\mathbf{1}\{Y_D \leq 2010\} \mid \text{ED}_m > 12]$	0.1541	0.1730
$\mathbb{E}[\mathbf{1}\{Y_D \leq 2010\} \mid \text{ED}_m \leq 12]$	0.1547	0.1676

Notes: These columns compare sample moments to moments calculated from model simulations performed using the MSM parameter estimates.

Table 4: Wage and Labor Moments - Data v. Model

Moment	Data	Model
$\mathbb{E}[h_{1,97}]$	6.1046	5.6888
$\mathbb{E}[h_{1,02}]$	6.1884	6.2471
$\mathbb{E}[h_{2,97}]$	4.1931	3.7914
$\mathbb{E}[h_{2,02}]$	4.3279	4.1858
$\mathbb{E}[\mathbf{1}\{w_{f,97} \leq \mathcal{W}_{f,0}\}]$	0.0999	0.1560
$\mathbb{E}[\mathbf{1}\{w_{f,97} \leq \mathcal{W}_{f,1}\}]$	0.4198	0.4028
$\mathbb{E}[\mathbf{1}\{w_{f,97} \leq \mathcal{W}_{f,2}\}]$	0.6561	0.6326
$\mathbb{E}[\mathbf{1}\{w_{f,97} \leq \mathcal{W}_{f,3}\}]$	0.7978	0.8046
$\mathbb{E}[\mathbf{1}\{w_{f,97} \leq \mathcal{W}_{f,4}\}]$	0.8918	0.9075
$\mathbb{E}[\mathbf{1}\{w_{f,97} \leq \mathcal{W}_{f,5}\}]$	0.9477	0.9623
$\mathbb{E}[\mathbf{1}\{w_{f,97} \leq \mathcal{W}_{f,6}\}]$	0.9830	0.9854
$\mathbb{E}[\mathbf{1}\{w_{m,97} \leq \mathcal{W}_{m,0}\}]$	0.0901	0.1388
$\mathbb{E}[\mathbf{1}\{w_{m,97} \leq \mathcal{W}_{m,1}\}]$	0.3480	0.3272
$\mathbb{E}[\mathbf{1}\{w_{m,97} \leq \mathcal{W}_{m,2}\}]$	0.5839	0.5525
$\mathbb{E}[\mathbf{1}\{w_{m,97} \leq \mathcal{W}_{m,3}\}]$	0.7596	0.7467
$\mathbb{E}[\mathbf{1}\{w_{m,97} \leq \mathcal{W}_{m,4}\}]$	0.8671	0.8788
$\mathbb{E}[\mathbf{1}\{w_{m,97} \leq \mathcal{W}_{m,5}\}]$	0.9268	0.9503
$\mathbb{E}[\mathbf{1}\{w_{m,97} \leq \mathcal{W}_{m,6}\}]$	0.9669	0.9820
$\mathbb{E}[w_{f,97}] \mid \text{ED}_f > 12]$	18.9728	20.1896
$\mathbb{E}[w_{f,02}] \mid \text{ED}_f > 12]$	23.2393	23.6535
$\mathbb{E}[w_{f,97}] \mid \text{ED}_f \leq 12]$	13.7708	17.7121
$\mathbb{E}[w_{f,02}] \mid \text{ED}_f \leq 12]$	16.3050	19.9115
$\mathbb{E}[w_{m,97}] \mid \text{ED}_m > 12]$	13.7227	15.1737
$\mathbb{E}[w_{m,02}] \mid \text{ED}_m > 12]$	16.2574	16.5423
$\mathbb{E}[w_{m,97}] \mid \text{ED}_m \leq 12]$	9.5932	12.2559
$\mathbb{E}[w_{m,02}] \mid \text{ED}_m \leq 12]$	11.6592	12.5806

Notes: These columns compare sample moments to moments calculated from simulations performed using the MSM parameter estimates.

Table 5A: Preference Parameter Estimates

Parameter	Estimate	St. Error
$\alpha_{1,1}$	0.7428	0.0532
$\alpha_{1,2}$	0.4156	0.1220
$\alpha_{1,3}$	0.0352	0.0044
$\alpha_{2,1}$	0.6823	0.0091
$\alpha_{2,2}$	0.3363	0.0988
$\alpha_{2,3}$	0.0575	0.0099
μ_{ζ}	6.5415	0.1212
σ_{ζ}	11.8043	1.0122
$\mu_{0.\theta}$	35.8103	1.0294
μ_{θ}^{1}	0.0032	0.0049
μ_{θ}^{2}	-0.0098	0.0034
ρ_{θ}	1.0629	0.1232
σ_{θ}	0.2511	0.0099

Table 5B: Wage Process Estimates

Parameter	Estimate	St. Error
$\mu_{w,1}$	2.1221	0.4084
$\mu_{w,2}$	2.2923	0.7645
$\sigma_{w,1}$	0.0921	0.0022
$\sigma_{w,2}$	0.0607	0.0019
$\Delta_{w,1,+}^{1}$	0.7666	0.0788
$\Delta_{w,1,-}^{1}$	0.0575	0.0099
$\Delta_{w,2,+}^{1}$	0.6672	0.0333
$\Delta_{w,2,-}^{1}$	0.1755	0.0998
$\Delta_{w,1,+}^{2}$	0.6926	0.0545
$\Delta_{w,1,-}^{2}$	0.0881	0.0016
$\Delta_{w,2,+}^{2}$	0.6863	0.0777
$\Delta_{w,2,-}^{2}$	0.1202	0.0232

Table 5C: Child Quality Production Function Estimates

Parameter	Estimate	St. Error
ψ_0	-0.0671	0.0111
ψ_d	-0.1398	0.0231
$\gamma_{\psi,a}$	0.0685	0.0129
$\gamma_{\psi,\theta}$	0.9659	0.0542
$\gamma_{\delta,0,1}$	0.1462	0.0332
$\gamma_{\delta,a,1}$	-0.0034	0.0001
$\gamma_{\delta,0,2}$	0.1181	0.0465
$\gamma_{\delta,a,2}$	-0.0070	0.0001
$\gamma_{\delta,0,k}$	1.0016	0.0013
$\gamma_{\delta,a,k}$	-0.0036	0.0001

Table 6: Divorce Standard Policy Experiment - Unilateral Divorce, Bilateral Divorce, and Divorce Ban

	Unilateral (Baseline)	Bilateral	No Divorce
Fertility Rate	0.5463	0.5733	0.5581
Divorce Rate	0.1190	0.0655	0.0000
Δ Test Scores	0.0000	-0.1007	-0.1054
% CEV Father	1.0000	1.0139	0.9323
% CEV Mother	1.0000	1.0428	1.0168

Notes: This table compares simulated fertility, divorce, child cognitive attainment, and welfare outcomes under the baseline unilateral divorce standard, a counterfactual bilateral divorce standard, and a divorce ban. The model includes no marriage choice, but couples may react to changing divorce standards through their fertility, investment, labor supply, and divorce choices. The bilateral divorce standard is implemented as described in Section 3. The divorce ban allows no divorce in any state of the world, whether or not spouses agree. Simulations are performed using the MSM parameter estimates reported in Table 5.

Table 7: Child Support Policy Experiment

	$\pi = 0.15$	$\pi = 0.3$	$\pi = 0.45$
Fertility Rate	0.5469	0.5508	0.5496
Divorce Rate	0.1204	0.1212	0.1214
Δ Test Scores	0.0000	-0.0092	-0.0167
% CEV Father	1.0000	1.0140	1.0216
% CEV Mother	1.0000	1.0074	1.0138

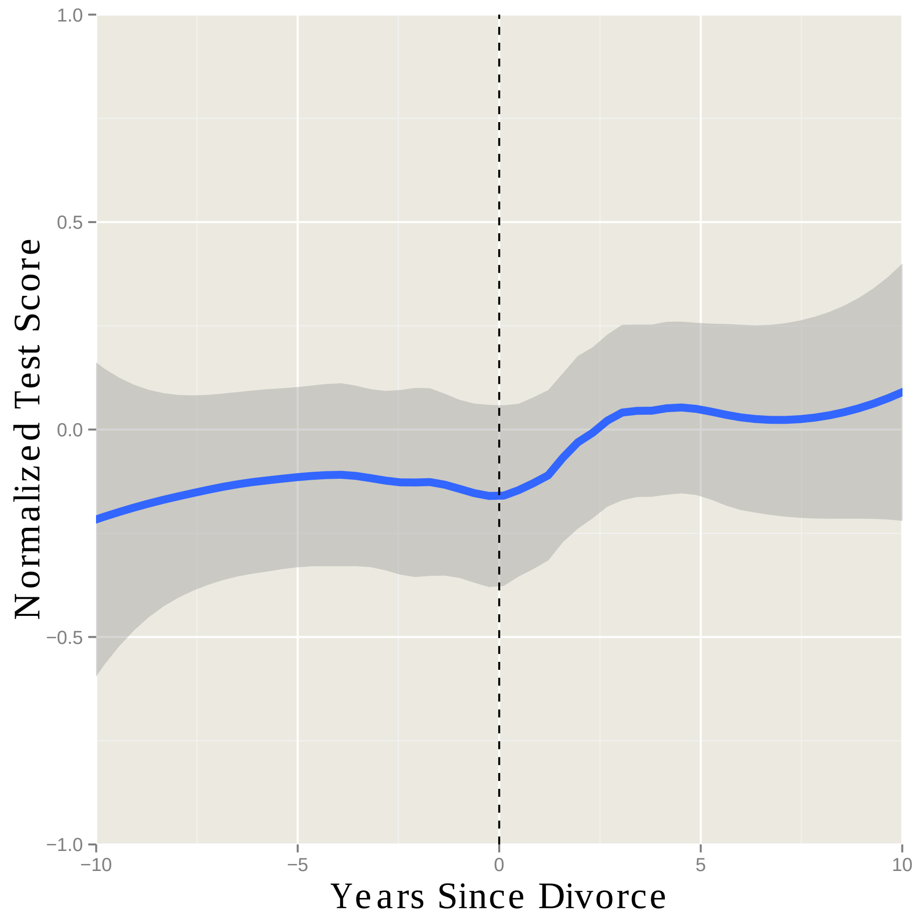
Notes: This table compares simulated fertility, divorce, child cognitive attainment, and welfare outcomes under the baseline 15 percent child support payment from father to mother in the event of divorce, and under counterfactual child support rates of 30 and 45 percent. The model includes no marriage choice, but couples may react to changing child support levels through their fertility, investment, labor supply, and divorce choices. The father is assumed to comply with the printed child support level. Simulations are performed using the MSM parameter estimates reported in Table 5.

Table 8: Decomposition of Skill Impacts

Policy	Marriage Qual. (Ψ)	Father's Time ($\mathcal{T}_1$)	Mother's Time ($\mathcal{T}_2$)	Initial Quality (k_0)
Divorce Standard	1.0891	-0.0324	-0.0567	0.0000
Child Support ($\pi = 0.3$)	1.1675	0.0467	-0.2144	0.0003

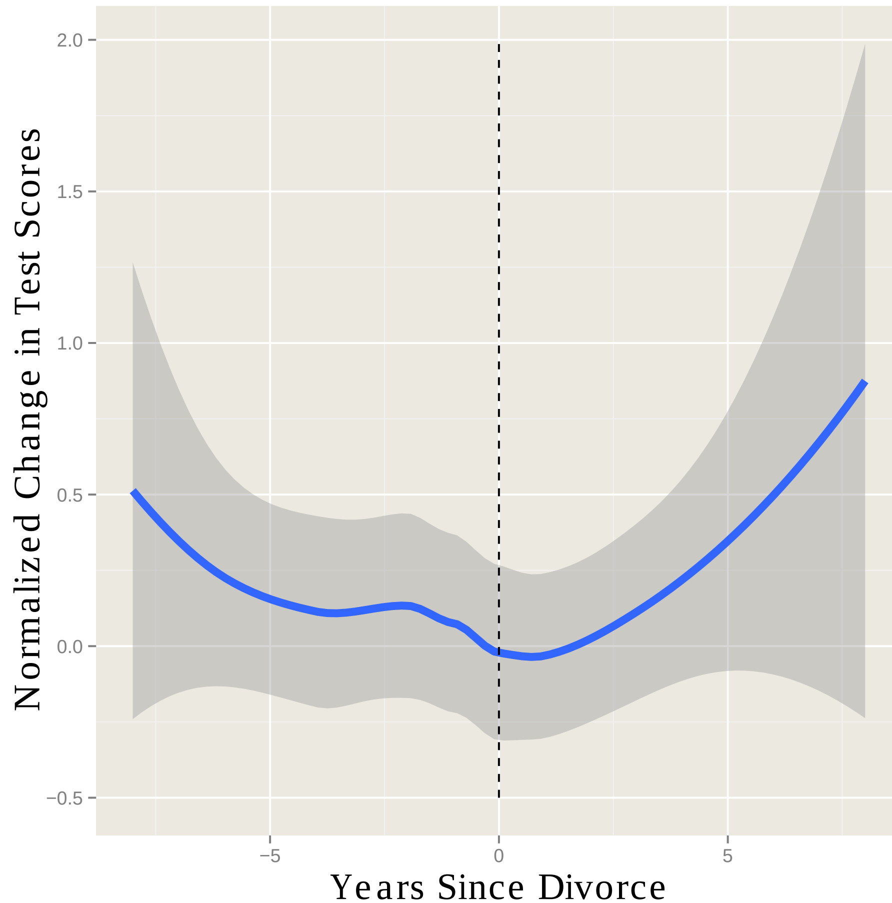
Notes: The entries in this table are computed following expression (21). The first row shows the decomposition of the change in average child quality in response to a move from a unilateral to a bilateral divorce standard into (i) the change arising from changing marriage quality, (ii) the change arising from the father's changing time investment, and (iii) the change arising from the mother's changing time investment. Similarly, the second row shows the decomposition of the change in average child quality in response to a move from the baseline child support rate ($\pi = 0.15$) to a higher rate ($\pi = 0.3$) into (i), (ii), and (iii). The contribution of each component is normalized by the total mean change in child cognitive skills.

Figure 1: Age-normed test score by years since divorce



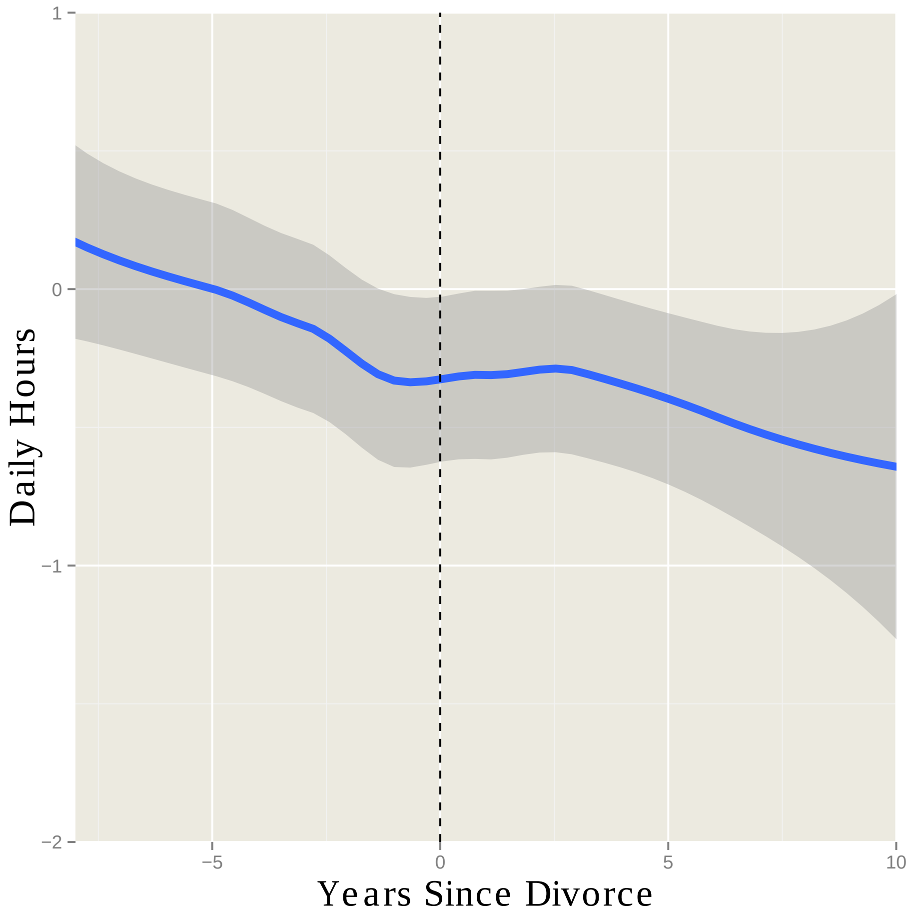
Notes: The figure displays the ormalized test scores by years to divorce. 10 = 10 years until divorce (at right), -10 = 10 years after divorce (at left). The values are locally smoothed means with 95% pointwise confidence intervals.

Figure 2: Growth from 1997 to 2002 in age-normed test score by years since divorce



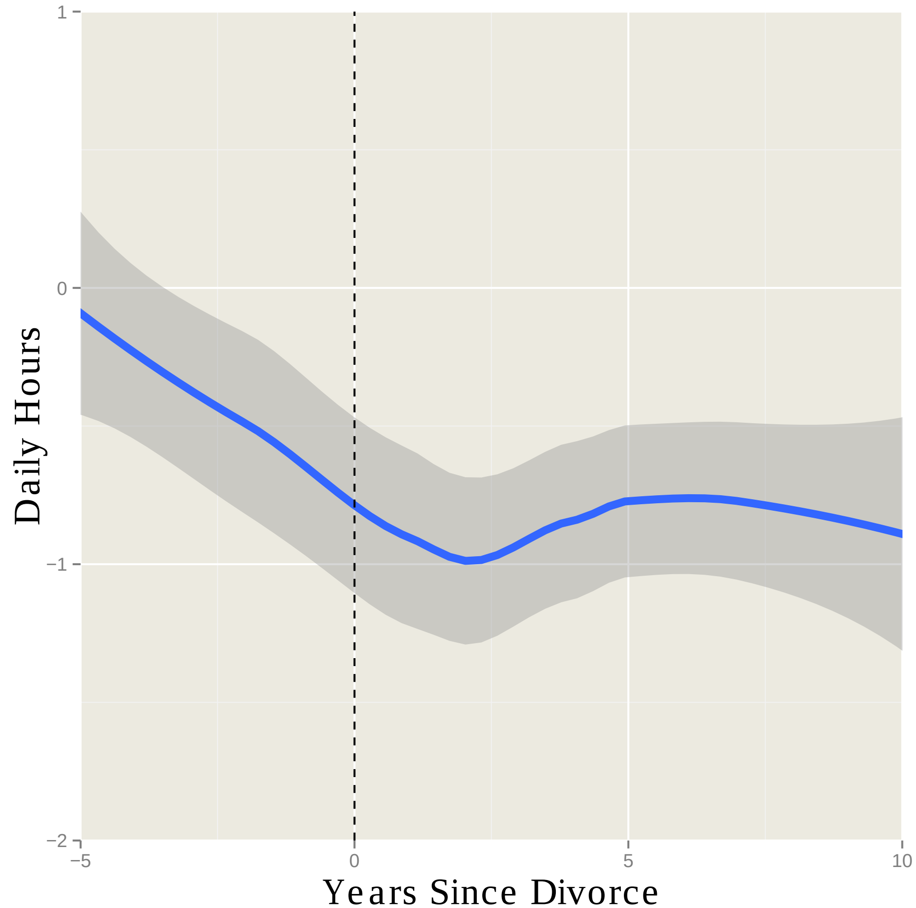
Notes: The figure displays the normalized growth in test scores by years to divorce. 10 = 10 years until divorce (at right), -10 = 10 years after divorce (at left). The values are locally smoothed means with 95% pointwise confidence intervals.

Figure 3: Mother's age-normed daily (active + passive) hours with the child by years since divorce



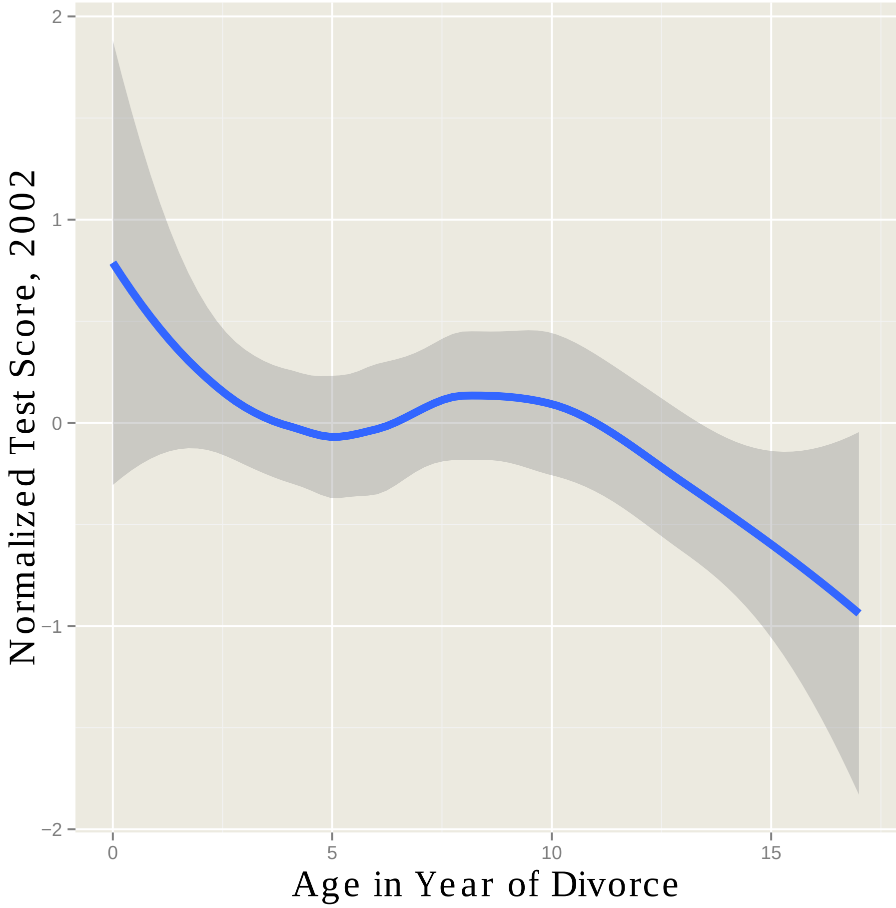
Notes: The figure displays the mother's daily hours with the child by years to divorce. 5 = 5 years until divorce (at right), -5 = 5 years after divorce (at left). The values are locally smoothed means with 95% pointwise confidence intervals.

Figure 4: Father's age-normed daily hours of (active + passive) time with the child by years since divorce



Notes: The figure displays the father's daily hours with the child by years to divorce. 5 = 5 years until divorce (at right), -5 = 5 years after divorce (at left). The values are locally smoothed means with 95% pointwise confidence intervals.

Figure 5: Age-normed test score in 2002 by age of child at parents' (past) divorce



Notes: The figure displays the child's normalized test score in 2002 child by the age of the child when the parents divorced. All child test scores contributing to this figure are observed after the date of divorce. The values are locally smoothed means with 95% pointwise confidence intervals.

Figure 6: Estimates of TFP Parameters: ψ_θ, ψ_d

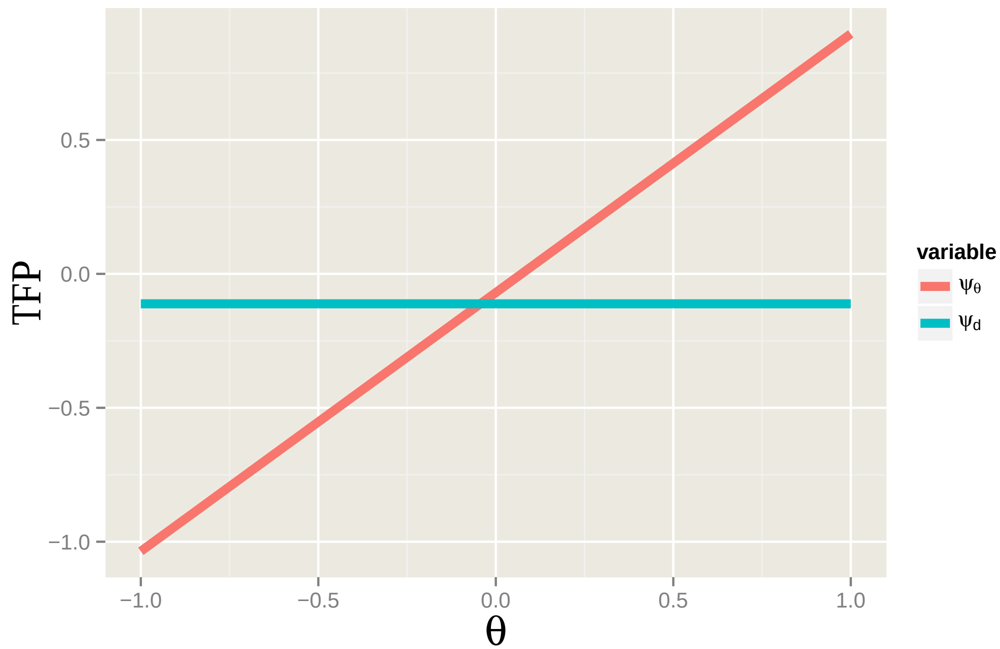


Figure 7: Estimates of Cobb-Douglas Shares: $\delta_{1,a}, \delta_{2,a}, \delta_{k,a}$

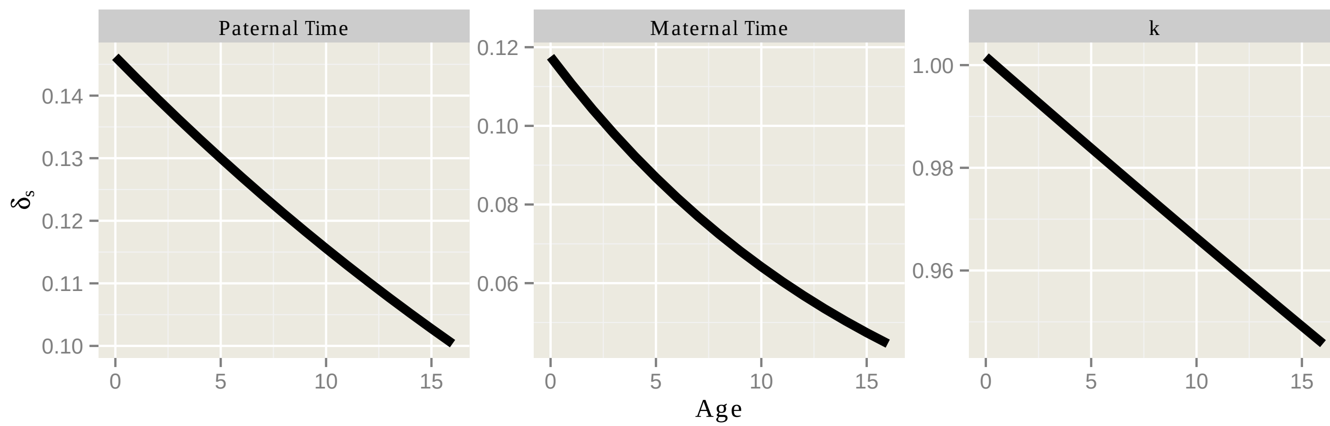


Figure 8: Father's Time Investment, $a = 0$

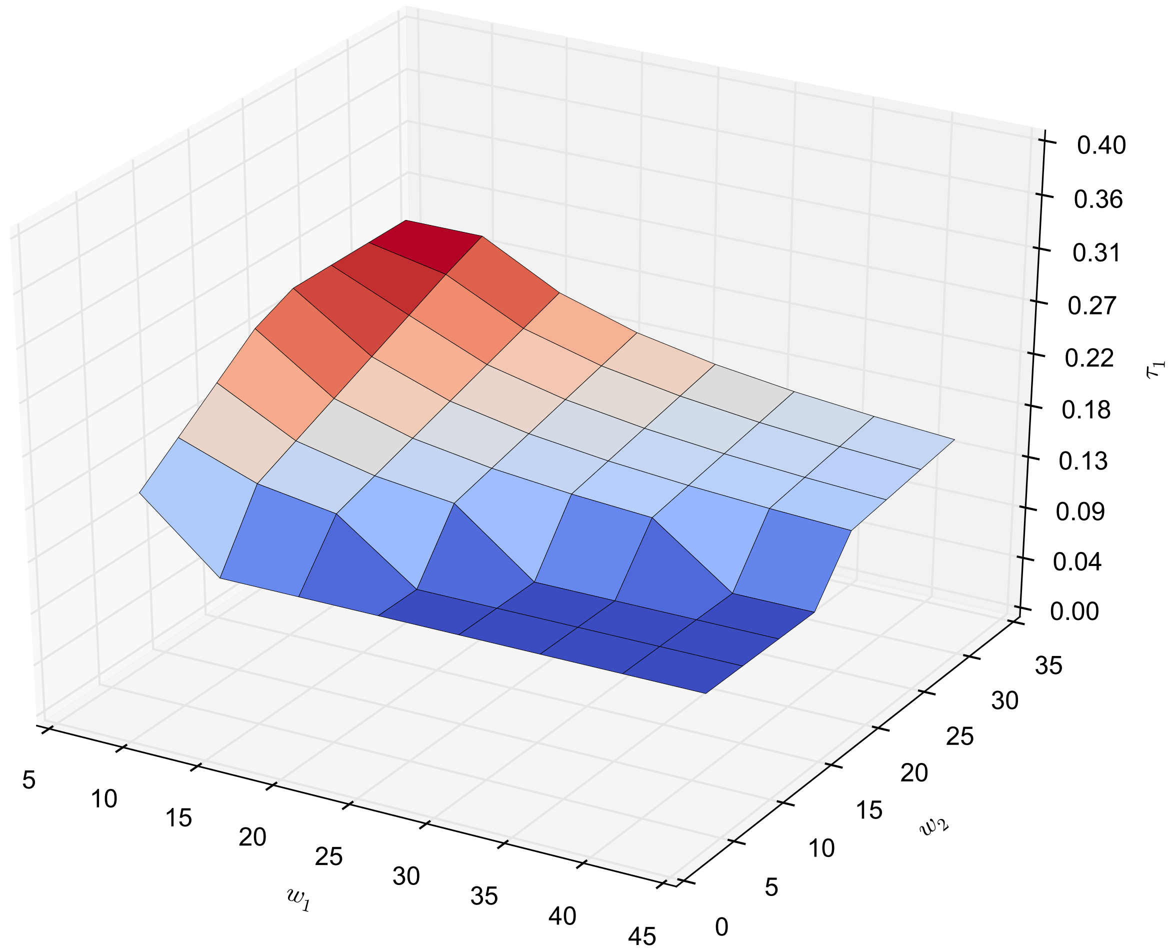


Figure 9: Divorce Probability, $a = 0, \theta = -1$

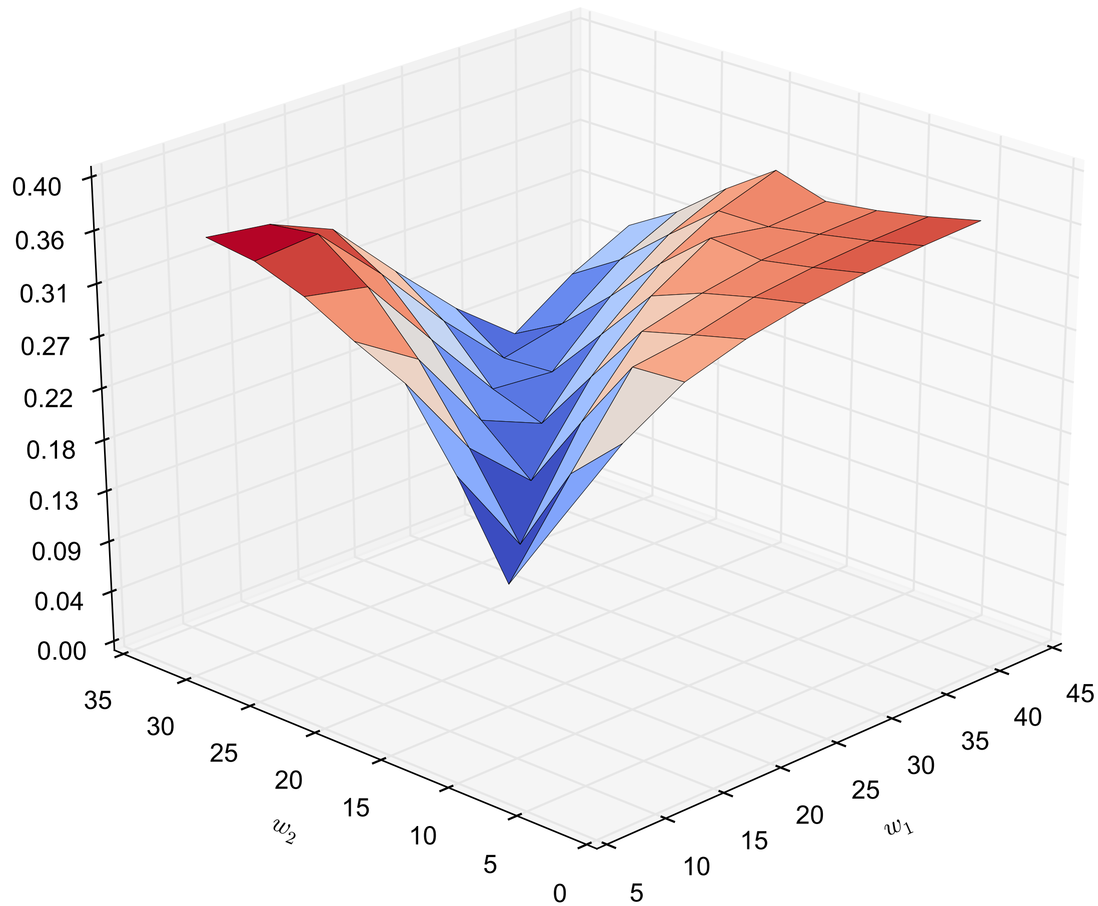
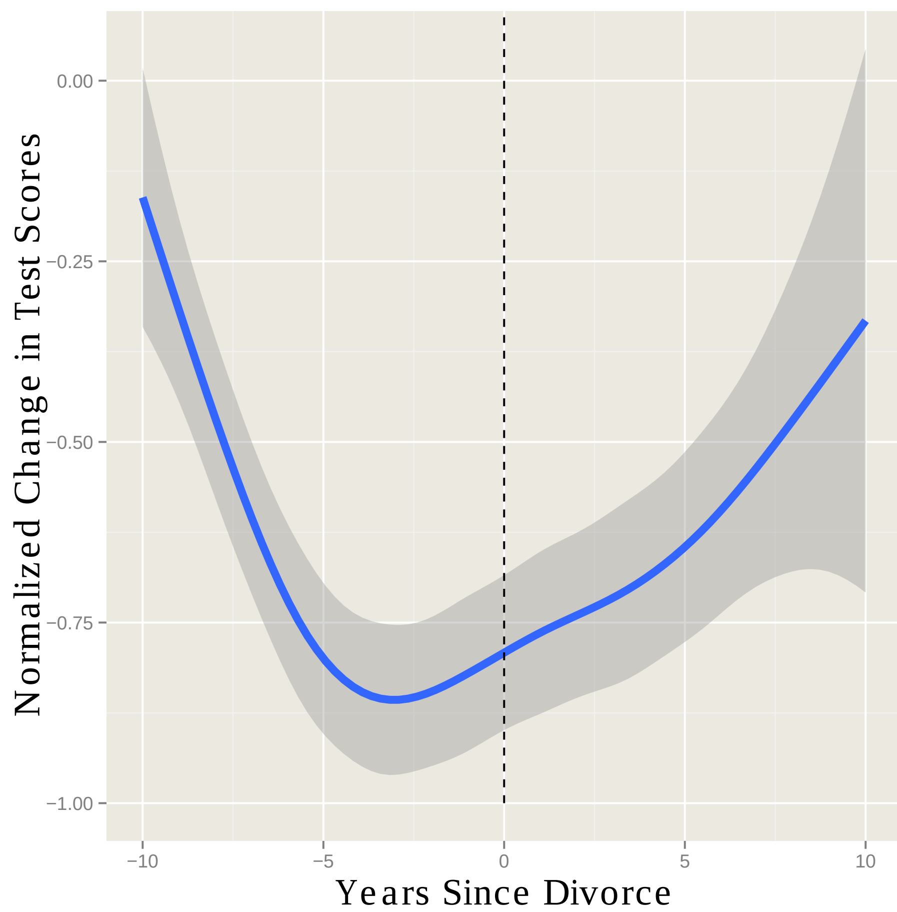
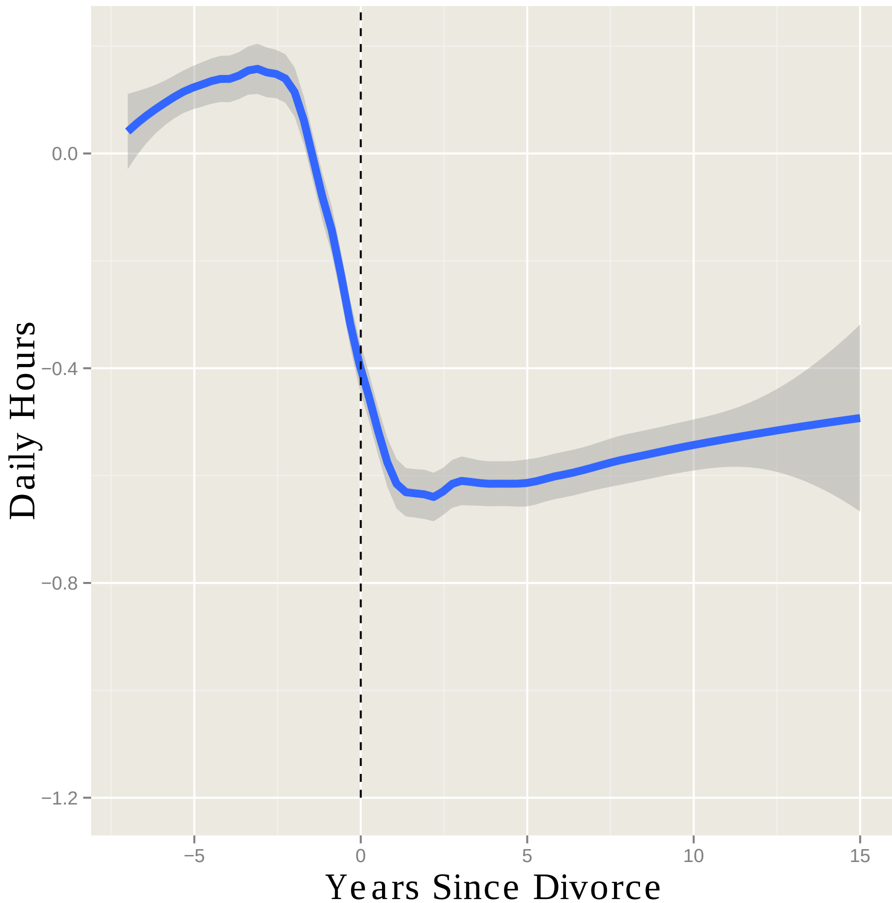


Figure 10: Model Normalized Test Score Growth by Years to Divorce



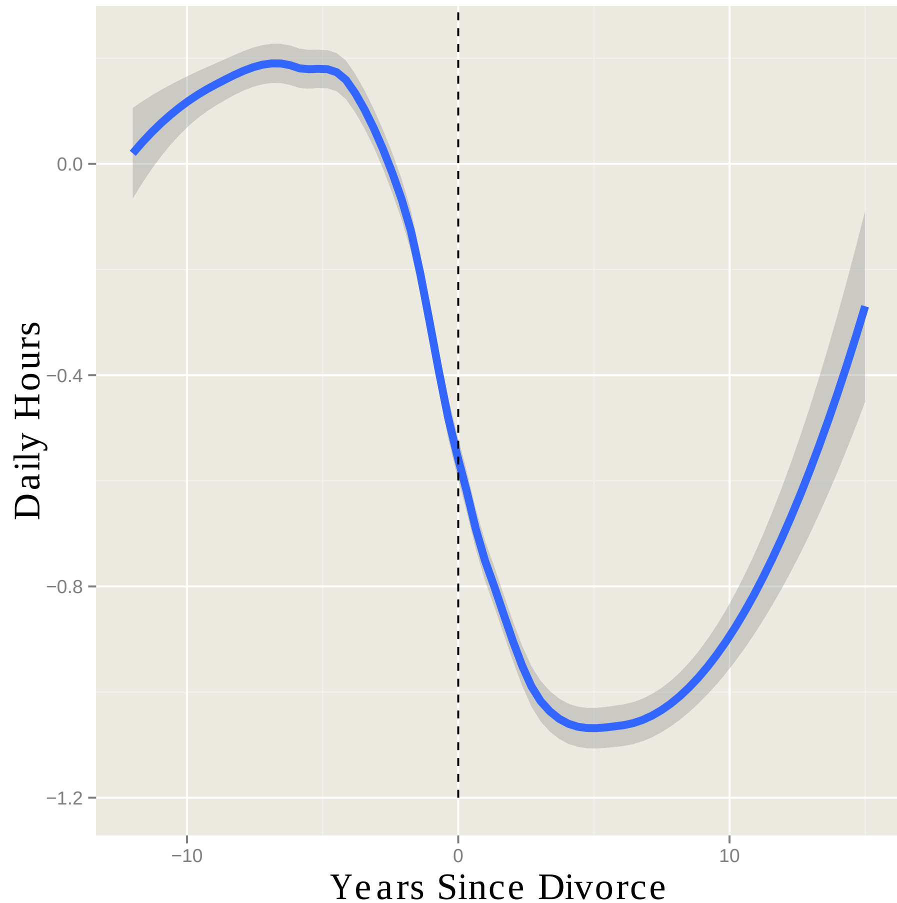
Notes: The figure displays the normalized growth in test scores by years to divorce. 10 = 10 years until divorce (at right), -10 = 10 years after divorce (at left). The values are locally smoothed means with 95% pointwise confidence intervals.

Figure 11: Model Normalized Paternal Time Input by Years Since Divorce



Notes: The figure displays the father's daily hours with the child by years to divorce. 15 = 15 years until divorce (at right), -5 = 5 years after divorce (at left). The values are locally smoothed means with 95% pointwise confidence intervals.

Figure 12: Model Normalized Maternal Time Input by Years Since Divorce



Notes: The figure displays the mother's daily hours with the child by years to divorce. 10 = 10 years until divorce (at right), -10 =10 years after divorce (at left). The values are locally smoothed means with 95% pointwise confidence intervals.

7 Details of the Model Solution

As a preliminary, recall that we express period utility as:

$$u_s(c, l, x) = \alpha_{1s} \log(c) + \alpha_{2s} \log(l) + \mathbf{1}\{a \geq 0\}(\alpha_{3s} \log(k) + \zeta) + (1 - \chi_d)\theta \qquad (22)$$

As we proceed through the following exposition, it will be convenient to write this expression as:

$$u_s(c, l, x) = \tilde{u}_s(c, l, \theta) + \mathbf{1}\{a \geq 0\}(\alpha_{3s} \log(k) + \zeta) \qquad (23)$$

Divorced at Maturity

When the child has reached maturity and the marriage has dissolved, parents choose only their private leisure and consumption. The state vector x can be written as $x = \{w_1, w_2, \emptyset, 17, k\}$ and the state next period can be written $\hat{x} = \{\hat{w}_1, \hat{w}_2, \emptyset, 17, k\}$. The value function can be written as:

$$V_s(x) = \max_{h_s} \{\alpha_{1s} \log(w_s h_s)) + \alpha_{2s} \log(1 - h_s) + \alpha_{3s} \log(k) + \zeta + \beta \mathbb{E}[V_s(\hat{x}) \mid x]\} \qquad (24)$$

Taking first order conditions, h_s has the closed form solution:

$$h_s = \frac{\alpha_{1s}}{\alpha_{1s} + \alpha_{2s}}. \qquad (25)$$

Thus, utility in each period is a closed form expression involving w_s only. The value function in this case can be solved simply as a matrix inversion problem using the transition matrix for wages, Π_{ws}. The value from children enters as an additive constant. Thus, we can write the value function as:

$$V_s(w_1, w_2, \emptyset, 17, k) = \frac{\alpha_{3s} \log(k) + \zeta}{1 - \beta} + \mathcal{V}_s(w_1, w_2, \emptyset, 17), . \qquad (26)$$

Let $z = \{w_1, w_2, \theta, a\}$ denote this reduced state space. We have that when $z = \{w_1, w_2, \emptyset, 17\}$, $\mathcal{V}_s(z)$ can be written recursively as:

$$\mathcal{V}_s(z) = \max_{h_s} \{\alpha_{1s} \log(w_s h_s) + \alpha_{2s} \log(1 - h_s) + \beta \mathbb{E}[\mathcal{V}_s(\hat{w}_1, \hat{w}_2, \emptyset, 17) \mid z]\} \qquad (27)$$

Thus, modulo an additive constant, the value function of divorced couples at maturity is the same as for those who divorce without children.

Married at Maturity

In this case, both spouses make labor supply decisions (h_1, h_2) and enjoy public consumption, as well as the utility derived from their adult child. The state in this stage can be written $x = \{w_1, w_2, \theta, 17, k\}$. We propose, as before, that the utility from children enters as a constant, writing:

$$V_s(x) = \frac{\alpha_{3s} \log(k) + \zeta}{1 - \beta} + \mathcal{V}_s(w_1, w_2, \theta, 17), \tag{28}$$

where:

$$\mathcal{V}_s(w_1, w_2, \theta, 17) = \max_{h_s} \left\{ \alpha_{1s} \log(c) + \alpha_{2s} \log(1 - h_s) + \beta \mathbb{E} \max_{d_s} \left[\mathcal{V}_s(\hat{z}) \mid d_1, d_2, z \right] \right\}, \tag{29}$$

subject to the constraints:

$$c = w_0 h_0 + w_1 h_1 \tag{30}$$

$$\hat{x} = \begin{cases} \{\hat{w}_1, \hat{w}_2, \emptyset, 17, k\} & \text{if } \mathbf{d}(d_1, d_1) = 1 \\ \{\hat{w}_1, \hat{w}_2, \hat{\theta}, 17\} & \text{if } \mathbf{d}(d_1, d_2) = 0 \end{cases} \tag{31}$$

Note the transition rule for divorce implies that divorce is unilateral. Divorce policies can be solved as the simple rule: $d_s(z) = \mathbf{1}\{\mathcal{V}(w_1, w_2, \emptyset, 17) \geq \mathcal{V}(w_1, w_2, \theta, 17)\}$. Taking first-order conditions, the equilibrium labor supply decisions are characterized as follows:

$$h_1 \geq 1 - \phi_1 - \phi_1 \left(\frac{w_2 h_2}{w_1} \right) \tag{32}$$

$$h_2 \geq 1 - \phi_2 - \phi_2 \left(\frac{w_1 h_1}{w_2} \right) \tag{33}$$

$$h_1 \geq 0 \tag{34}$$

$$h_2 \geq 0 \tag{35}$$

$$\phi_s = \frac{\alpha_{2s}}{\alpha_{1s} + \alpha_{2s}} \tag{36}$$

Uniqueness of this solution is simple to verify. We can solve by first checking each corner condition, then solving for the internal solution. The interior solution is:

$$h_1 = \frac{1 - \phi_1 - \phi_1 \frac{w_2}{w_1}(1 - \phi_2)}{1 + \phi_1 \phi_2} \tag{37}$$

$$h_2 = \frac{1 - \phi_2 - \phi_2 \frac{w_1}{w_2}(1 - \phi_1)}{1 + \phi_2 \phi_1} \tag{38}$$

As before, the value from the child enters as an additive constant. The other component of the value function can be solved either by value function iteration or gradient based solution. Thus, we see that additive separability of the value function in $\log(k)$ is preserved by the recursion.

Divorced during Development

So far, the value function has been additively separable in terms of child quality; a component that is linear in $\log(k)$. We will show that this property holds as we move backwards through each stage of the parental investment problem. This provides a huge simplification of the model's solution. First, we state the problem. Then, we'll show that the property holds in the penultimate period before maturity. This should be sufficient to demonstrate that it holds in all previous periods, as well as to show the recursion that defines the coefficents on $\log(k)$ each period.

Mother's Problem

We start with the mother's problem. To do this, we take the father's choices, (h_1, τ_1), as given. The value function at $a = 16$ can be written:

$$V_2(w_1, w_2, \emptyset, 16, k) = \max_{h_2,\tau_2} \left\{ \tilde{u}(c, 1 - h_2 - \tau_2, \theta) + \alpha_{23} \log(k) + \beta \mathbb{E}[V_2(\hat{w}_1, \hat{w}_2, \emptyset, 17, \hat{k})] \right\} \quad (39)$$

$$\hat{k} = \psi_{16}(0) \tau_1^{\delta_{1,16}} \tau_2^{\delta_{2,16}} k^{\delta_{k,16}} \quad (40)$$

$$c = \pi w_1 h_1 + w_2 h_2 \quad (41)$$

$$h_2 \geq 0 \quad (42)$$

$$\tau_2 \leq \overline{\tau}_2 \quad (43)$$

Note the introduction of two divorce law parameters. $\overline{\tau}_2 = 1 - \overline{\tau}_1$ sets the allocation of custody time, while π is the marginal tax rate on the Father's earnings that is transferred to the Mother.

Now, substituting the form for V_s given in (26) and using the production function (3), we get

$$\begin{aligned} V_2(w_1, w_2, \emptyset, 16, k) = \max_{h_2,\tau_2} \Bigg\{ & \tilde{u}(c, 1 - h_2 - \tau_2) + \alpha_{23} \log(k) \\ & + \frac{\beta \alpha_{23}}{1 - \beta} [\log(\psi_{16}(0)) + \delta_{1,16} \log(\tau_1) + \delta_{2,16} \log(\tau_2) + \delta_{k,16} \log(k)] + \beta \mathbb{E}[\mathcal{V}_2(\hat{z}) \mid z] \Bigg\} \end{aligned} \quad (44)$$

Finally, letting $x = \{w_1, w_2, \emptyset, 16, k\}$, $z = \{w_1, w_2, \emptyset, 16\}$, and collecting terms gives the following:

$$V_2(x) = \alpha_{V,2,16} \log(k) + \mathcal{V}_{md,2}(z) \quad (45)$$

$$\alpha_{V,2,16} = \alpha_{23} + \beta \delta_{k,16} \frac{\alpha_{23}}{1 - \beta} \quad (46)$$

$$\mathcal{V}_2(z) = \max_{h_2,\tau_2} \left\{ \tilde{u}(c, 1 - h_2 - \tau_2) + \frac{\beta \alpha_{23}}{1 - \beta} [\delta_{1,16} \log(\tau_1) + \delta_{2,16} \log(\tau_2)] + \beta \mathbb{E}[\mathcal{V}_s(\hat{w}_1, \hat{w}_2, \emptyset, 17)] \right\} \quad (47)$$

So we see that the additive structure is preserved in this case. By induction then, we can write the value function at age a as:

$$V_{fd,2}(w,k,a) = \mathcal{V}_{fd,2}(w,a) + \alpha_{V,2,a}\log(k) \quad (48)$$

$$\alpha_{V,2,a} = \alpha_{23} + \beta\delta_{k,a}\alpha_{V,2,a+1} \quad (49)$$

$$\mathcal{V}_2(z) = \max_{h_2,\tau_2}\{\tilde{u}(c, 1-h_2-\tau_2) + \beta\mathbb{E}[\mathcal{V}_2(\hat{z}) + \alpha_{V,2,a+1}(\log(\psi_a(0)) + \delta_1\log(\tau_1) + \delta_2\log(\tau_2)) \mid z]\} \quad (50)$$

$$\hat{z} = \{\hat{w}_1, \hat{w}_2, \emptyset, a+1\} \quad (51)$$

$$c = \pi w_1 h_1 + w_2 h_2 \quad (52)$$

$$h_2 \geq 0 \quad (53)$$

$$\tau_2 \leq \overline{\tau}_2 \quad (54)$$

These decisions are made taking the father's choices of (h_1, τ_1) as given. The first order conditions yield

$$h_2 = \max\left\{\frac{\alpha_{21}}{\alpha_{21}+\alpha_{22}}(1-\tau_2) - \frac{\alpha_{22}}{\alpha_{21}+\alpha_{22}}\frac{\pi h_1 w_1}{w_2}, 0\right\} \quad (55)$$

$$\tau_2 = \min\left\{\frac{\delta_2\beta\alpha_{2,V,t+1}}{\alpha_{22}+\delta_2\beta\alpha_{2,V,t+1}}(1-h_2), \overline{\tau}_2\right\} \quad (56)$$

Given h_1, we can solve the system above by solving for h_2 assuming that the constraint on τ_1 does not bind:

$$h_2 = \max\left\{\frac{\alpha_{21}}{\alpha_{21}+\alpha_{22}+\delta_2\beta\alpha_{V,2,t+1}} - \frac{\alpha_{22}+\delta_2\beta\alpha_{V,2,t+1}}{\alpha_{21}+\alpha_{22}+\delta_2\beta\alpha_{V,2,t+1}}\frac{\pi h_1 w_1}{w_2}, 0\right\} \quad (57)$$

If the corresponding solution to τ_2 violates the constraint, then we set $\tau_2 = \overline{\tau}_2$ and solve:

$$h_2 = \max\left\{\frac{\alpha_{21}}{\alpha_{21}+\alpha_{22}}(1-\overline{\tau}_2) - \frac{\alpha_{22}}{\alpha_{21}+\alpha_{22}}\frac{\pi h_1 w_1}{w_2}, 0\right\} \quad (58)$$

This solution method is valid since we know only one interior solution exists. Thus the constraint must bind if it is violated by this single interior solution.

Father's Problem

Logic when solving for the Father's problem is identical. Therefore we skip verification of additive separability and state the problem in this separable form:

$$\mathcal{V}_1(z) = \max_{h_1,\tau_1}\{\tilde{u}(c, 1-h_1-\tau_1) + \beta\mathbb{E}[\mathcal{V}_1(\hat{z}) + \alpha_{V,1,a+1}(\log(\psi_a(0)) + \delta_1\log(\tau_1) + \delta_2\log(\tau_2)) \mid z]\} \quad (59)$$

$$\alpha_{V,1,a} = \alpha_{13} + \beta\delta_{k,a}\alpha_{V,1,a+1} \quad (60)$$

$$c = (1-\pi)w_1 h_1 \quad (61)$$

$$h_1 \geq 0 \quad (62)$$

$$\tau_1 \leq \overline{\tau}_1 \quad (63)$$

The father's decisions are made taking the mother's choice of τ_2 as given. His first order conditions yield:

$$h_1 = \frac{\alpha_{11}}{\alpha_{11}+\alpha_{12}}(1-\tau_1) \quad (64)$$

$$\tau_1 = \min\left\{\overline{\tau}_1, \frac{\delta_1\beta\alpha_{1,V,t+1}}{\alpha_{12}+\delta_1\beta\alpha_{1,V,t+1}}(1-h_1)\right\} \quad (65)$$

The above can be rearranged to yield

$$\tau_1 = \min\left\{\overline{\tau}_1, \frac{\delta_1\beta\alpha_{1,V,t+1}}{\alpha_{11}+\alpha_{12}+\delta_1\beta\alpha_{1,V,t+1}}\right\}. \quad (66)$$

Married during Development

Once again, we wish to show that the additive separability of the value function is preserved when married. We will do this for the mother in the final period of development, which should be sufficient to establish the intuition. Once again, take the labor supply, investment, and divorce decisions of the husband, (h_1, τ_1, d_1), as given. The value function at $a = 16$ can be written:

$$V_2(w_1, w_2, \theta, 16, k) = \max_{\tau_2, l_2, h_2}\left\{\tilde{u}_2(c, l_2, \theta) + \alpha_{23}\log(k) + \beta\mathbb{E}\max_{d_2}\left[V_2(\hat{x}) \mid d_1, d_2, \tau_1, \tau_2, x\right]\right\} \quad (67)$$

$$c = w_0 h_0 + w_1 h_1 \quad (68)$$

$$\hat{x} = \begin{cases} \{\hat{w}_1, \hat{w}_2, \emptyset, 17, \hat{k}\} & \text{if } \mathbf{d}(d_1, d_1) = 1 \\ \{\hat{w}_1, \hat{w}_2, \hat{\theta}, 17, \hat{k}\} & \text{if } \mathbf{d}(d_1, d_2) = 0 \end{cases} \quad (69)$$

$$\hat{k} = \psi_{16}(\theta)\tau_1^{\delta_{1,16}}\tau_2^{\delta_{2,16}}k^{\delta_{k,16}} \quad (70)$$

$$\tau_s + h_s + l_s = 1 \quad (71)$$

Now, substituting the form for V_2 given in (26) and (28), and using the production function (3), we get

$$V_2(w_1, w_2, \theta, 16, k) = \max_{h_2, \tau_2}\left\{\tilde{u}(c, 1-h_2-\tau_2) + \alpha_{23}\log(k) + \frac{\beta\alpha_{23}}{1-\beta}[\log(\psi_{16}(\theta)) + \delta_{1,16}\log(\tau_1) + \delta_{2,16}\log(\tau_2) + \delta_{k,16}\log(k)] + \beta\mathbb{E}[\mathcal{V}_2(\hat{z}) \mid d_1, d_2, z]\right\} \quad (72)$$

with

$$\hat{z} = \begin{cases} \{\hat{w}_1, \hat{w}_2, \emptyset, 17\} & \text{if } \mathbf{d}(d_1, d_1) = 1 \\ \{\hat{w}_1, \hat{w}_2, \hat{\theta}, 17\} & \text{if } \mathbf{d}(d_1, d_2) = 0 \end{cases}$$

The additional step in the case of marriage requires us to verify that child quality k does not affect the divorce decision. Inspecting equations (26) and (28) we see that:

$$V_s(w_1, w_2, \emptyset, a, k) \geq V_s(w_1, w_2, \theta, a, k) \Leftrightarrow \mathcal{V}_s(w_1, w_2, \emptyset, a) \geq \mathcal{V}_s(w_1, w_2, \theta, a) \tag{73}$$

This follows by the fact that the coefficient on $\log(k)$, $(1-\beta)^{-1}[\alpha_{s3} + \zeta]$, is unaffected by marital status. Thus, finally, we can collect terms and write:

$$V_2(x) = \alpha_{V,2,16} \log(k) + \mathcal{V}_2(z) \tag{74}$$

$$\alpha_{V,2,16} = \alpha_{23} + \beta\delta_{k,16}\frac{\alpha_{23}}{1-\beta} \tag{75}$$

$$\tag{76}$$

And the value function $\mathcal{V}_2$ can be written as:

$$\mathcal{V}_2(w_1, w_2, \theta, 16) = \max_{h_2, \tau_2}\Bigg\{\tilde{u}(c, 1-h_2-\tau_2, \theta) + \frac{\beta\alpha_{23}}{1-\beta}[\log(\psi_{16}(\theta)) + \delta_{1,16}\log(\tau_1) + \delta_{2,16}\log(\tau_2)] + \beta\mathbb{E}[\mathcal{V}_2(\hat{z}) \mid d_1, d_2, z]\Bigg\}, \tag{77}$$

with

$$\hat{z} = \begin{cases} \{\hat{w}_1, \hat{w}_2, \emptyset, 17\} & \text{if } \mathbf{d}(d_1, d_1) = 1 \\ \{\hat{w}_1, \hat{w}_2, \hat{\theta}, 17\} & \text{if } \mathbf{d}(d_1, d_2) = 0 \end{cases}$$

Importantly, note that the coefficients on $\log(k)$ are the same as in the case for divorce. This is crucial since it implies that the divorce decision is unaffected by child quality. This logic is preserved by further induction, hence we can write the marriage problem more generally as:

$$V_2(x) = \alpha_{V,2,a} \log(k) + \mathcal{V}_2(z) \tag{78}$$

Where $\mathcal{V}_{fm,2}$ is given by:

$$\mathcal{V}_2(w_1, w_2, \theta, a) = \max_{h_2, \tau_2}\Big\{\tilde{u}(c, 1-h_2-\tau_2, \theta) + \beta\alpha_{2,V,a+1}[\log(\psi_a(\theta)) + \delta_{1,a}\log(\tau_1) + \delta_{2,a}\log(\tau_2)] + \beta\mathbb{E}[\mathcal{V}_2(\hat{z}) \mid z]\Big\}, \tag{79}$$

subject to:

$$c = w_1 h_1 + w_2 h_2 \tag{80}$$

$$h_2 \geq 0 \tag{81}$$

$$\hat{z} = \begin{cases} \{\hat{w}_1, \hat{w}_2, \emptyset, a+1\} & \text{if } \mathbf{d}(d_1, d_1) = 1 \\ \{\hat{w}_1, \hat{w}_2, \hat{\theta}, a+1\} & \text{if } \mathbf{d}(d_1, d_2) = 0 \end{cases} \tag{82}$$

$$\tag{83}$$

In similar fashion to the marriage game at maturity, the mother's solution can be written as:

$$h_2 = \max\left\{\frac{\alpha_{21}}{\alpha_{21}+\alpha_{22}+\delta_2\beta\alpha_{V,2,t+1}} - \frac{\alpha_{22}+\delta_2\beta\alpha_{V,2,a+1}}{\alpha_{21}+\alpha_{22}+\delta_2\beta\alpha_{V,2,a+1}}\frac{h_1 w_1}{w_2}, 0\right\} \tag{84}$$

$$\tau_2 = \frac{\delta_2\beta\alpha_{V,2,a+1}}{\alpha_{22}+\delta_2\beta\alpha_{V,2,t+1}}(1-h_2) \tag{85}$$

The father's solution and the expression for $\mathcal{V}_1$ is symmetric. Note that the solution to this game can be computed in an identical fashion to the marriage game at maturity. We have the interior solution:

$$h_1 = \frac{1-\phi_1-\phi_1\frac{w_2}{w_1}(1-\phi_2)}{1+\phi_1\phi_2} \tag{86}$$

$$h_2 = \frac{1-\phi_2-\phi_2\frac{w_1}{w_2}(1-\phi_1)}{1+\phi_2\phi_1} \tag{87}$$

$$\phi_s = \frac{\alpha_{s2}+\delta_s\beta\alpha_{V,s,a+1}}{\alpha_{s1}+\alpha_{s2}+\delta_2\beta\alpha_{V,s,a+1}} \tag{88}$$

We are free to let the production parameters vary by marriage quality.